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Abstract:
We formulate a general cointegrated vector autoregressive (CVAR) model that nests both a class of consumption Euler equations and various Keynesian type consumption functions. Using likelihood-based methods and Norwegian data, we find support for cointegration between consumption, income and wealth once a structural break around the financial crisis is allowed for. That consumption cointegrates with both income and wealth and not only with income points to the empirical irrelevance of an Euler equation. Moreover, we find that consumption equilibrium corrects to changes in income and wealth and not that income equilibrium corrects to changes in consumption, which would be the case if an Euler equation is true. We also find that most of the parameters stemming from the class of Euler equations are not corroborated by the data when considering conditional expectations of future consumption and income in CVAR models. Only habit formation seems important in explaining the Norwegian consumer behaviour. Our preferred model is a dynamic Keynesian type consumption function with a first year marginal propensity to consume out of income close to 25 per cent.

Keywords: Consumption Euler equation, Keynesian consumption function, financial crisis, structural break, conditional expectations

JEL classification: C51, C52, E21

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Sammendrag

Vi formulerer en generell kointegrerende vektor autoregressiv (KVAR) modell som nøster både en klasse av Euler-ligninger for konsum og ulike konsumfunksjoner basert på keynesiansk teori. Basert på sannsynlighetsmaksimeringsmetoder og norske data finner vi støtte for kointegrasjon mellom husholdningenes konsum, inntekt og formue når et strukturelt brudd rundt finanskrisen i 2008 er tatt hensyn til. At konsum kointegrerer med både inntekt og formue, og ikke bare med inntekt, er ikke i tråd med en Euler-ligning for konsum. Våre analyser viser også at konsum likevektsjusterer ved endringer i inntekt og formue og ikke at inntekt likevektsjusterer ved endringer i konsum, som ville være tilfelle dersom en Euler-ligning fant støtte i data. Vi finner heller ingen støtte for de fleste parametrene som stammer fra klassen av Euler-ligninger i KVAR modeller med betingede forventninger for konsum og inntekt. Bare vanekonsum, som er forenlig med en Euler-ligning, synes å bidra til å forklare den norske konsumatferden. Vår foretrukne empiriske modell er en dynamisk keynesiansk konsumfunksjon med en første års marginal konsumtilbøyelighet ved endring i inntekt på rundt 25 prosent.

1 Introduction

Economists have for a long time been concerned with how households react to changes in fiscal policy. The financial crisis in 2008 led to renewed interest in how household asset composition, liquidity and credit market conditions may affect consumption, see for instance Muellbauer (2010, 2016) and Kaplan *et al.* (2018). As a response to the financial crisis many governments used expansionary fiscal policies, but at the expense of increasing public and private debt levels in later periods. Expansionary fiscal policies were soon followed by contractionary policies in the wake of the financial crisis in many countries. The effects of these fiscal policies depend on the marginal propensity to consume (MPC) out of shocks to income. In the economics literature there was until recently no consensus regarding the size of the MPC and the role of fiscal policy in stabilizing the economy was controversial. A new consensus seems now to be emerging on the size of the MPC that is much larger than what used to be common in many DSGE models. For instance, the heterogeneity-augmented model by Carroll *et al.* (2017) predicts an aggregate MPC of around 20 per cent compared to roughly 5 per cent implied by the standard macroeconomic models with representative agents.

In contrast to the Keynesian consumption function, which says that changes in current household income affect consumption markedly, both the permanent income hypothesis by Friedman (1957) and the life-cycle hypothesis by Ando and Modigliani (1963) imply that consumption depends on unanticipated and not on anticipated income shocks with a much stronger response to permanent than transitory shocks. These hypotheses are typically formulated as consumption Euler equations where the representative agent is a permanent income consumer that does not respond much to transitory income changes. Recent microeconometric studies, however, find that households react much stronger to transitory income shocks than what the standard forward-looking theory of consumption predicts. For instance, Jappelli and Pistaferri (2014) estimate an average MPC of 48 per cent using Italian data and Fagereng *et al.* (2016) find an MPC of 35 per cent using Norwegian data. Also, studies of anticipated tax cuts find larger responses to consumption (excess sensitivity) than what is expected from the forward-looking theory of consumption, see for instance Parker *et al.* (2013).

Extended versions of the standard forward-looking theory that allow for precautionary savings, liquidity constraints and habit formation can explain some of the empirical results found in the literature. Campbell and Mankiw (1991) among others account for precautionary savings and liquidity constraints in a model for aggregate consumption assuming constant relative risk aversion (CRRA) utility preferences and some of the households being current income consumers. Likewise, Yogo (2004) and Canzoneri *et al.* (2007) find a small response of aggregate consumption to changes in the real interest rate after controlling for income. Carroll *et al.* (1992), on the other hand, assumes that consumers who face income uncertainty and are both impatient and prudent behave according to the so-called buffer-stock theory of saving. According to this theory unemployment expectations may explain parts of household behaviour because unpredictable fluctuations in income caused by spells of unemployment are an important source of uncertainty facing many households even in countries where replacement ratios are quite high.[1] Deaton (1991) presents another version of the buffer-stock model based on income uncertainty and liquidity constraints where households use liquid assets to buffer against temporary income shocks. Kaplan and Violante (2014) introduce trading costs to explain evidence of current income consumers even for those who are wealthy due to illiquid assets and credit constraints. The consumption model by Smets and Wouters (2003), which many DSGE models typically are based upon, includes habit formation in that current consumption is proportional to past consumption.

Our contributions of the present paper are threefold. First, we formulate a general cointegrated vector autoregressive (CVAR) model that nests both a class of consumption Euler equations and various Keynesian type consumption functions. The former include

[1] The replacement ratio is defined as the proportion of benefits received when unemployed against income levels when employed. In Norway, the replacement ratio for average wage earners is 62 per cent.

a version of the martingale hypothesis by Hall (1978) and the equations of precautionary savings and liquidity constraints as in Campbell and Mankiw (1991) and of habit formation as in Smets and Wouters (2003). Using likelihood methods, one can test the properties of *cointegration* between consumption and income only and of *equilibrium correction* in the nesting CVAR. Drawing upon Eitrheim *et al.* (2002), the former property represents the common ground for a Keynesian type consumption function and a consumption Euler equation and the latter represents the discriminating feature between them.

Then, we consider conditional expectations of future consumption and income in CVAR models within the context of Johansen and Swensen (1999, 2004, 2008). Since as pointed out by Tinsley (2002) "empirical rejection of rational expectations is the rule rather than the exception in macroeconomics", we note the possibility to use the strategy of dividing the parameters of well fitted CVAR models into two parts: parameters of interest which are the parameters describing rational expectations and nuisance parameters which are the parameters necessary to ensure satisfactory empirical fit. By this strategy it is possible to focus on economically interesting parameters stemming from the class of Euler equations. Our treatment of the role of conditional expectations of future consumption and income is quite similar to what has been done in the new Keynesian literature on pricing behaviour, see Boug *et al.* (2010, 2017).

Finally, we study aggregate Norwegian consumer behaviour, both before and after the financial crisis. Using seasonally unadjusted quarterly data that span the period from the early 1980s to the end of 2016, we find support for cointegration between consumption, income and wealth once a structural break around the financial crisis in 2008 is allowed for. Our finding that consumption cointegrates with both income and wealth and not only with income is evidence against an Euler equation of consumer behaviour. Likelihood ratio tests further show that consumption equilibrium corrects to changes in income and wealth and not that income equilibrium corrects to changes in consumption, as would be the case when an Euler equation is true. We also find that most of the parameters stemming from the class of Euler equations are not corroborated by the data when considering conditional expectations of future consumption and income in CVAR models. Only habit formation in accordance with Smets and Wouters (2003) seems to play an important role in explaining the Norwegian consumer behaviour. Our preferred model is a dynamic Keynesian type consumption function that has a first year MPC of around 25 per cent, which is in line with the empirical findings in the recent literature.

The rest of the paper is structured as follows: Section 2 discusses the theoretical background and how the various hypotheses of consumer behaviour are nested within a general CVAR. Section 3 presents the data used in the empirical analysis. Section 4 reports findings from the cointegration analysis. Section 5 presents results from considering conditional expectations of consumption and income in CVAR models. Section 6 provides a conclusion.

2 Theoretical background

As a useful benchmark for the empirical analysis, we begin this section by outlining the martingale hypothesis derived by Hall (1978). Then, we present the often used consumption Euler equations with precautionary savings, liquidity constraints and habit formation based on CRRA utility preferences. Finally, we formulate a general CVAR that nests the various hypotheses from the set of Euler equations as well as Keynesian type consumption functions.

2.1 The martingale hypothesis

Since the seminal paper by Hall (1978) the martingale hypothesis, saying that no other variable than consumption at time t should help predict consumption at time $t+1$, has been subject to extensive empirical investigation, see for instance Flavin (1981), Campbell and Deaton (1989), Muellbauer and Lattimore (1995), Palumbo *et al.* (2006) and Muellbauer (2010).

The main idea behind the martingale hypothesis, which builds on the permanent income hypothesis by Friedman (1957) and the life cycle hypothesis by Ando and Modigliani (1963), is that the representative consumer bases the choice between consumption and savings on both initial financial wealth, current income and prospects of future income. Formally, the consumer solves the following intertemporal optimisation problem under uncertainty:[2]

$$\max E_t \sum_{i=0}^{\infty} (1+\theta)^{-i} U(C_{t+i}) \tag{1}$$

subject to

$$W_{t+1} = (1+R_t)(W_t + YL_t - C_t) \tag{2}$$

and

$$\lim_{i \to \infty} E_t \left[W_{t+i}/(1+R_t)^i \right] = 0, \tag{3}$$

where E_t, θ, $U(\cdot)$ and C_{t+i} in (1) denote expectations conditional on information at time t, the subjective discount rate, assumed constant, the utility function, assumed additive over time, and consumption at time $t+i$, respectively. Thus, the consumer maximises the present discounted value of expected utility conditional on information at time t subject to the budget constraint in (2) and the No-Ponzi Game condition in (3), where W_t denotes financial wealth at time t, YL_t is labour income at time t and R_t denotes the riskless rate of real return at time t.

The well known first order condition or the Euler equation for this optimisation prob-

[2] We follow Blanchard and Fischer (1989, p. 279) here. Our exposition differs slightly, however, in that we use an infinite time approach and assume that savings by the consumer can only be invested in riskless assets, which essentially are bank deposits in practice. Although the riskless rate of real return may vary over time, we further assume that it can be treated as non-stochastic at time t.

lem takes the form

$$U'(C_t) = (1 + R_t)(1 + \theta)^{-1} E_t U'(C_{t+1}), \tag{4}$$

where $U'(C_t)$ is marginal utility at time t. Assuming that the utility function is quadratic and that the riskless rate of real return is constant and equal to the subjective discount rate, cf. Hall (1978), (4) becomes

$$E_t C_{t+1} = C_t, \tag{5}$$

or $\Delta C_{t+1} = \varepsilon_{t+1}$, where $E_t \varepsilon_{t+1} = 0$, that is ε_{t+1} is an unforecastable innovation in permanent income. Hence, consumption follows a martingale, which means that the consumer never plans to change the consumption level from one period to the next. The change in consumption is thus unforecastable. Also, (5) implies the familiar property of certainty equivalence such that no precautionary savings is undertaken by the consumer. Using the result in (5) together with the constraints in (2) and (3), we obtain

$$C_t = R(1 + R)^{-1} W_t + R(1 + R)^{-1} \sum_{i=0}^{\infty} (1 + R)^{-i} E_t Y L_{t+i} \equiv Y P_t, \tag{6}$$

which says that optimal consumption equals the sum of the proceeds from financial wealth and the expected present value of future labour income, defined to equal the permanent income YP_t. Finally, (6) and (2) imply that

$$\Delta C_t = R(1 + R)^{-1} \sum_{i=0}^{\infty} (1 + R)^{-i} (E_t - E_{t-1}) Y L_{t+i} \equiv \Delta Y P_t. \tag{7}$$

Any change in consumption, ΔC_t, is equal to the annuity value of the revisions in expectations from the last period to the present one about current and future labour income, defined to equal the change in permanent income, $\Delta Y P_t$. This implication is consistent with (5) as any change in consumption must be founded in new and unexpected information about what the consumer can afford. No other factors change consumption over time. If we for instance assume that labour income follows a stationary first order autoregressive process with coefficient $0 < \rho < 1$, (7) becomes $\Delta C_t = R(1 + R - \rho)^{-1}\epsilon_t$, where ϵ_t represents an unexpected change or innovation in labour income from period $t - 1$ to t. Accordingly, the marginal propensity to consume in response to an unexpected change in labour income is given by $R(1 + R - \rho)^{-1}$, which is close to R since $R - \rho$ is likely to be small. Consumption is thus smoother than transitory changes in labour income. When ρ approaches unity the marginal propensity to consume also approaches unity and unexpected labour income is hardly smoothed at all.

A useful alternative formulation of the forward looking theory of consumption, suggested by Campbell (1987), is the so-called "saving for a rainy day" hypothesis, which says that

$$S_t = -\sum_{i=1}^{\infty}(1 + R)^{-i} E_t \Delta Y L_{t+i}, \tag{8}$$

where $S_t \equiv Y_t - C_t$ and $Y_t \equiv R(1 + R)^{-1}W_t + Y L_t$. Hence, savings equal the expected discounted value of future declines in labour income. That is, the consumer "saves for a rainy day". As shown by Campbell and Deaton (1989) and used by Palumbo *et al.* (2006) among others, (8) has a very similar form in logarithms. By approximating the saving ratio, $\frac{S_t}{YL_t}$, with the logarithms of the income to consumption ratio, $y_t - c_t$, we may write a logarithmic version of (8) as[3]

$$y_t - c_t \approx -\sum_{i=1}^{\infty} \rho^i E_t \Delta y l_{t+i} + \varsigma, \tag{9}$$

where ρ and ς denote a discount factor and a constant, respectively. Campbell and Deaton (1989, equation 8) discuss conditions under which (9) is an approximation to (8). Equation

[3]Here and below lower case letters denote the logarithms of a variable.

(9) says that the saving ratio and the expected future labour income growth are negatively related so that savings are increasing today when the consumer anticipates income to decline tomorrow. We also note that if labour income is non-stationary, $I(1)$, the saving ratio is stationary, $I(0)$, and income and consumption are cointegrated with a coefficient equal to unity. In Subsection 2.3, this property of cointegration in the "saving for a rainy day" hypothesis will be explored in a nesting CVAR.

We have seen that quadratic preferences lead to certainty equivalence with the consequence that an increase in uncertainty faced by the consumer has no effect on consumption and savings. Moreover, the forward looking model above relies heavily on the assumption of perfect capital markets and does not include habit formation. To allow for precautionary savings, liquidity constraints and habit formation, we now turn to consumption Euler equations with CRRA preferences.

2.2 Euler equations with CRRA preferences

Whereas Blundell and Stoker (2005) consider heterogeneity across consumers with CRRA preferences, we simplify matters following Campbell and Mankiw (1991) and Smets and Wouters (2003) among others and assume that all consumers are identical with respect to marginal utility and willingness to move consumption from one period to another. Our point of departure, as in Campbell and Mankiw (1991), is a CRRA utility function of the form[4]

$$U(C_t) = (1-\delta)^{-1} C_t^{1-\delta} \text{ for } 1 \neq \delta > 0, \tag{10}$$

where δ is the inverse of the intertemporal elasticity of substitution, σ. The Euler equation now becomes

$$E_t C_{t+1}^{-\delta} = (1+\theta)(1+R_t)^{-1} C_t^{-\delta}, \tag{11}$$

[4] $U(C_t) = \ln C_t$ for $\delta = 1$.

or $E_t[\exp(-\delta \ln C_{t+1})] = (1+\theta)(1+R_t)^{-1}\exp(-\delta \ln C_t)$. Unlike Campbell and Mankiw (1991), who allow for ex ante real interest rates to vary over time, we simplify matters further by considering ex post real interest rates in (11). Assuming that the logarithms of consumption is normally distributed with mean $E_t \ln C_{t+1}$ and time varying variance η_{t+1}^2, and making use of the approximation $\ln[(1+\theta)(1+R_t)^{-1}] \cong \theta - R_t$, we may write the Euler equation as

$$E_t \Delta c_{t+1} = \frac{\eta_{t+1}^2}{2\sigma} - \sigma\theta + \sigma R_t, \tag{12}$$

or $\Delta c_{t+1} = \frac{\eta_{t+1}^2}{2\sigma} - \sigma\theta + \sigma R_t + \varepsilon_{t+1}$, where $E_t \varepsilon_{t+1} = 0$ is the expectation of an innovation term, ε_{t+1}, in permanent income. Under our assumptions the variance of ε_{t+1} equals η_{t+1}^2. Clearly, if the consumer faces more uncertainty, that is a larger η_{t+1}^2, consumption is expected to increase from this period to the next. Thus, the consumer reduces consumption now in response to increased uncertainty to have a larger safety buffer, that is precautionary savings, for more consumption in the next period. As pointed out by Blundell and Stoker (2005), consumption growth with precautionary savings generally depends on the conditional variance of the uninsurable components of innovations to income. When the variance, η^2, is constant, (12) simplifies to

$$E_t \Delta c_{t+1} = \phi + \sigma R_t, \tag{13}$$

or $\Delta c_{t+1} = \phi + \sigma R_t + \varepsilon_{t+1}$, where the constant term, $\phi = \frac{\eta^2}{2\sigma} - \sigma\theta$, partly reflects precautionary savings. According to (13), savings by the consumer is also associated with intertemporal substitution in consumption. An increase in the real interest rate makes savings more profitable due to relatively costly consumption today, hence consumption is expected to increase from this period to the next.

The underlying assumption that the consumer has access to perfect capital markets in the sense of no liquidity constraints, permits consumption to move freely in accordance with (13). In practice, however, the consumer may be credit rationed by lending criteria based on payment-to-income ratios, which prevents the consumer from acting in accordance with the forward-looking hypothesis. To account for liquidity constraints in a simple way, Campbell and Mankiw (1991) assume that aggregate consumption is equal to a weighted average with weights μ and $1-\mu$ reflecting the proportions of rule of thumb consumers and permanent income consumers, respectively. Campbell and Mankiw (1991) further assume that rule of thumb consumers determine consumption growth as a weighted average of current and one period lag of income growth with weights λ and $1-\lambda$.[5] We can then formulate an augmented version of (13) as

$$E_t \Delta c_{t+1} = (1-\mu)\phi + \mu[\lambda E_t \Delta y_{t+1} + (1-\lambda)\Delta y_t] + (1-\mu)\sigma R_t, \tag{14}$$

or $\Delta c_{t+1} = (1-\mu)\phi + \mu[\lambda \Delta y_{t+1} + (1-\lambda)\Delta y_t] + (1-\mu)\sigma R_t + (1-\mu)\varepsilon_{t+1}$, where Δy_{t+1} and Δy_t are disposable income growth at time $t+1$ and t. When $\mu = 0$, the augmented model collapses to (13). As emphasised by Campbell and Mankiw (1991), (14) can only serve as an approximation to a model in which liquidity constraints are explicitly modelled. Moreover, as stressed by Basu and Kimball (2002) and later by Gali *et al.* (2007), the interpretation of the results in Campbell and Mankiw (1991) hinges on the assumption of utility preferences that are separable in consumption and labour (leisure). Otherwise, due to high correlation between changes in disposable income and hours worked, a significant μ may be the outcome from estimation of (14) even if all consumers are fully permanent income consumers. Nevertheless, a fully worked out model with liquidity constraints involves more complicated consumer behaviour, see for instance Deaton (1992, p. 194-213) and Blundell and Stoker (2005).

We may also formulate an augmented specification of (14) by adding lagged change

[5] Campbell and Deaton (1989) argue that consumption is smooth because it responds with a lag to changes in income. As pointed out by Campbell and Mankiw (1991), (14) with lagged income growth is also in the spirit of Flavin (1981).

in consumption, Δc_t, and an equilibrium correction term, $(c_t - \nu y_t)$, such that

$$(15) \quad E_t \Delta c_{t+1} = (1-\mu)\phi + \mu[\lambda E_t \Delta y_{t+1} + (1-\lambda)\Delta y_t] + (1-\mu)\sigma R_t + \tau \Delta c_t + \varrho(c_t - \nu y_t),$$

where consumption and income are cointegrated with the parameter ν. As pointed out by Campbell and Mankiw (1991), Δc_t would appear in (15) with $\tau > 0$ if there are important quadratic adjustment costs in consumption whereas $(c_t - \nu y_t)$ would appear with $\varrho < 0$ in a disequilibrium model of consumption and income.[6]

The consumption Euler equation by Smets and Wouters (2003), typically included in DSGE models, is also based on CRRA preferences appearing in a utility function separable in consumption and labour. However, the marginal utility of consumption at time t now equals $\epsilon_t(C_t - hC_{t-1})^{-\delta}$, where ϵ_t and hC_{t-1} denote, respectively, a shock to the subjective discount rate that affects the intertemporal substitution and a habit formation that is proportional to past consumption.[7] Hence, Smets and Wouters (2003) extend the Euler equation in (11) by taking into account the possibility of habit formation. To obtain a tractable empirical model, Smets and Wouters (2003) log-linearize the Euler equation around a non-stochastic steady state such that consumption obeys

$$(16) \qquad c_t = (1-\omega_1)c_{t-1} + \omega_1 E_t c_{t+1} - \omega_2 \hat{r}_t,$$

where $\omega_1 = (1+h)^{-1}$, $\omega_2 = \frac{(1-h)}{(1+h)\delta}$ and $\hat{r}_t$ is the log deviation of the ex ante real interest rate from its non-stochastic steady state.[8] Consumption thus depends on a weighted average of past and expected future consumption and the ex ante real interest rate. The higher the degree of habit formation, the smaller is the impact of the real interest rate on consumption

[6] Campbell and Mankiw (1991) impose $\nu = 1$ and find both factors to be insignificant for a number of countries. However, for the UK the rejection of the equilibrium correction term is contested by one commentator, see Hendry (1991).

[7] Note that $\delta = \sigma_c$ in Smets and Wouters (2003).

[8] We simplify matters by disregarding shocks from ε_t in (16). Note also that the log deviation of the consumption level from its non-stochastic steady state, $\ln(C/\bar{C})$, and the homogeneity restriction between the past and the future consumption levels imply that $\ln\bar{C}$ cancels throughout in (16).

for a given elasticity of substitution. We note that (16) collapses to $E_t \Delta c_{t+1} = \sigma \hat{r}_t$ when $h = 0$, which essentially is the same as (13). Adding and subtracting c_{t-1} and $\omega_1 E_t c_t$ on the right hand side of (16) and rearranging, we can write expected consumption growth when $h \neq 0$ as

$$E_t \Delta c_{t+1} = \varpi_1 \Delta c_t + \varpi_2 \hat{r}_t, \tag{17}$$

where $\varpi_1 = \frac{1-\omega_1}{\omega_1}$ and $\varpi_2 = \frac{\omega_2}{\omega_1}$. Hence, effects on expected consumption growth of lagged change in consumption can either be attributed to habit formation, as in (17), or to quadratic adjustment costs in consumption, as in (15). We add as a final remark that in some DSGE models, in the spirit of Campbell and Mankiw (1991), rule of thumb consumers are incorporated into the model, see for instance Amato and Laubach (2003) and Di Bartolomeo and Rossi (2007) and the references cited therein.

2.3 A nesting CVAR

Thus far we have focused on various consumption models based on Euler equations. There exists, however, a huge empirical literature initiated by Davidson *et al.* (1978) based on a rather different theoretical framework, which goes back to Keynes (1936), saying that current aggregate income is an important determinant of current aggregate consumption.

The consumption models employed by Brodin and Nymoen (1992), Eitrheim *et al.* (2002), Erlandsen and Nymoen (2008) and Jansen (2013), which are all based on Norwegian data, belong to this literature. These studies have in common a Keynesian type consumption function of the form

$$c_t = \beta_y y_t + \beta_w w_t, \tag{18}$$

where c_t, y_t and w_t denote real consumption, real disposable income and real household wealth, respectively. Assuming that c_t, y_t and w_t are integrated series of order one, I(1), (18) implies cointegration between the three variables with the cointegration parameters β_y and β_w for income and wealth. As pointed out by Brodin and Nymoen (1992), (18) represents one and only one cointegrating vector in the case of homogeneity between consumption, income and wealth, that is $\beta_w = 1 - \beta_y$. Both Erlandsen and Nymoen (2008) and Jansen (2013) augment (18) by the real after tax interest rate as a separate variable to capture the possibility of long run substitution effects in consumption. An increase in the real after tax interest rate is assumed to make consumption today more expensive relative to consumption tomorrow. Hence, consumption is expected to decline. Interestingly, (18) and the "saving for a rainy day" hypothesis in (9) share the same cointegration property between consumption and income in the special case when $\beta_y = 1$ and $\beta_w = 0$.

We are now ready to formulate a general CVAR that nests all the Euler equations considered above as well as the various Keynesian type consumption functions inherent in (18). To show this, we will draw upon the analysis by Eitrheim *et al.* (2002) stating that both the Euler equation approach and the Keynesian consumption function approach are consistent with cointegration between consumption and income and that the discriminating feature is their implications for the direction of equilibrium correction (weak exogeneity) in a CVAR.

As opposed to Jansen (2013), who considers partial CVAR models in which the ex post real after tax interest rate is conditioned upon at the outset,[9] we start out with a *full* CVAR representation of a p-dimensional VAR of order k written as

$$\Delta X_t = \Pi X_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta X_{t-j} + \gamma t + \vartheta + \Phi D_t + \epsilon_t, \tag{19}$$

where Δ is the difference operator, $X_t = (c_t, y_t, w_t, R_t)'$ comprises real consumption, c_t, real

[9] Jansen (2013) also conditioned upon an age composition variable at the outset because, as documented in Erlandsen and Nymoen (2008), aggregate consumption may rise and savings decrease when the share of elderly persons increases in the population. However, we do not include the age composition variable in the CVAR as it turned out to be insignificant in Jansen (2013).

disposable income, y_t, real household wealth, w_t, and the real after tax interest rate, R_t, as the modelled variables, t is a deterministic trend, Γ_j and Φ are matrices of coefficients, γ is a vector of coefficients, ϑ is a vector of intercepts, D_t is a vector of centered seasonal dummies, and ϵ_t are normally distributed random variables with expectation zero and unrestricted covariance matrix Ω. The initial observations $X_1, \ldots, X_k$ are considered as given. The impact matrix Π has rank $0 \leq r \leq p$, and therefore can be written $\Pi = \alpha\beta'$, where α and β are $p \times r$ matrices of adjustment coefficients and cointegration coefficients, respectively, of full rank r.

The Euler equation approach implies that consumption, wealth and the real after tax interest rate are *not* equilibrium correcting and that income alone, in line with the "savings for a rainy day" hypothesis in (9), *is* equilibrium correcting. These properties and the various hypotheses considered in Subsections 2.1 and 2.2 are, as we shall see, nested in the CVAR when $r = 2$. By leading (19) one period and taking the conditional expectations of ΔX_{t+1}, we can write out the CVAR when $k = 2$ for notational simplicity as

$$(20) \quad E_t \begin{pmatrix} \Delta c_{t+1} \\ \Delta y_{t+1} \\ \Delta w_{t+1} \\ \Delta R_{t+1} \end{pmatrix} = \begin{pmatrix} \alpha_{c1} & \alpha_{c2} \\ \alpha_{y1} & \alpha_{y2} \\ \alpha_{w1} & \alpha_{w2} \\ \alpha_{R1} & \alpha_{R2} \end{pmatrix} \begin{pmatrix} 1 & \beta_{y1} & 0 & \beta_{R1} \\ -1 & 1 & \beta_{w2} & \beta_{R2} \end{pmatrix} \begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \end{pmatrix}$$
$$+ \begin{pmatrix} \gamma_{1,11} & \gamma_{1,12} & \gamma_{1,13} & \gamma_{1,14} \\ \gamma_{1,21} & \gamma_{1,22} & \gamma_{1,23} & \gamma_{1,24} \\ \gamma_{1,31} & \gamma_{1,32} & \gamma_{1,33} & \gamma_{1,34} \\ \gamma_{1,41} & \gamma_{1,42} & \gamma_{1,43} & \gamma_{1,44} \end{pmatrix} \begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix} + \gamma t + \vartheta + \Phi D_{t+1},$$

where $E_t\epsilon_{t+1} = 0$, $\beta_{y1} = -\nu$ from (15) and exact identification of the two cointegrating vectors is achieved by imposing $\beta_{c1} = 1$ and $\beta_{w1} = 0$ in the first row of β' and $\beta_{c2} = -1$ and $\beta_{y2} = 1$ in the second row of β', all dictated from the theory of cointegration between consumption and income. The consumption Euler equation and the "saving for a rainy day"hypothesis together impose $\beta_{y1} = -1$ and $\beta_{w2} = \alpha_{w1} = \alpha_{w2} = \alpha_{R1} = \alpha_{R2} = 0$ as additional restrictions on the cointegrating part of (20), which makes the two cointegrating vectors not identifiable. Still the system in (20) provides important insights by deriving some of the single equation relationships in Subsection 2.2 from it.

We note in particular that consumption is *not* equilibrium correcting only when $\alpha_{c1} = \alpha_{c2}$ and that this restriction can be tested empirically once the two cointegrating vectors are *exactly* identified. When $\alpha_{c1} = \alpha_{c2}$ the consumption Euler equation in the case of no rule of thumb consumers is given by $E_t\Delta c_{t+1} = \vartheta_c + \alpha_{c1}(\beta_{R1} + \beta_{R2})R_t$, where $\Gamma_1 = 0$, $\gamma = 0$, $\Phi = 0$, $\vartheta_c = \phi$ and $\alpha_{c1}(\beta_{R1} + \beta_{R2}) = \sigma$, in accordance with (13).

The "saving for a rainy day" hypothesis is likewise given by $E_t\Delta y_{t+1} = \vartheta_y + (\alpha_{y1} - \alpha_{y2})(c_t - y_t) + (\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})R_t$, where $\vartheta_y = \kappa$ and $(\alpha_{y1} - \alpha_{y2})^{-1} = \rho$, in line with (9). The additional term $(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})R_t$ states the "savings for a rainy day" hypothesis in (20) somewhat less restrictive than (9) in the sense that the real after tax interest rate is allowed to vary over time. The additional term is easy to handle such that the CVAR also nests all the hypotheses in (15) with some rule of thumb consumers. To see this, we multiply (20) by the matrix $c' = (1, -\mu\lambda, 0, 0)$ and rearrange terms to obtain the following version of (15):

$$
\begin{aligned}
(21) \qquad E_t\Delta c_{t+1} - \mu\lambda E_t\Delta y_{t+1} \;&=\; \vartheta_c - \mu\lambda\vartheta_y + (\gamma_{1,12} - \mu\lambda\gamma_{1,22})\Delta y_t \\
&\quad + [\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})]R_t \\
&\quad + (\gamma_{1,11} - \mu\lambda\gamma_{1,21})\Delta c_t - \mu\lambda(\alpha_{y1} - \alpha_{y2})(c_t - y_t),
\end{aligned}
$$

where $\gamma = 0$, $\Phi = 0$, $\vartheta_c - \mu\lambda\vartheta_y = (1-\mu)\phi$, $\gamma_{1,12} - \mu\lambda\gamma_{1,22} = \mu(1-\lambda)$, $\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2}) = (1-\mu)\sigma$, $\gamma_{1,11} - \mu\lambda\gamma_{1,21} = \tau$ and $-\mu\lambda(\alpha_{y1} - \alpha_{y2}) = \varrho$. The theories we have discussed above entail different outcomes for subsequent empirical estimation of the consumption equation.

- First, a modified version of the martingale hypothesis by Hall (1978), $E_t \Delta c_{t+1} = 0$,[10] implies that $\mu\lambda$ equals zero and that no significant terms appear on the right hand side of (21).

- Second, precautionary savings in response to uncertainty are reflected in the intercept, $\vartheta_c - \mu\lambda\vartheta_y$.

- Third, a significantly positive estimate of $[\alpha_{c1}(\beta_{R1} + \beta_{R2}) - \mu\lambda(\alpha_{y1}\beta_{R1} + \alpha_{y2}\beta_{R2})]$ can be interpreted as the intertemporal elasticity of substitution in consumption.

- Fourth, a significantly positive estimate of $(\gamma_{1,11} - \mu\lambda\gamma_{1,21})$ points to quadratic adjustment costs or habit formation in consumption.

- Fifth, significantly positive estimates of $\mu\lambda$ and $(\gamma_{1,12} - \mu\lambda\gamma_{1,22})$ indicate a substantial portion of rule of thumb consumers responding to current and one period lag in income growth, respectively.

- Finally, a significantly positive estimate of $\mu\lambda(\alpha_{y1} - \alpha_{y2})$ can be interpreted as the coefficient of speed of adjustment in a disequilibrium model of consumption and income.

The Keynesian consumption function approach, as opposed to the Euler equation approach, implies that consumption *is* equilibrium correcting in the CVAR. To simplify the exposition, we now assume that $r = 1$. When the cointegration vector is normalised with respect to consumption and $k = 2$, the CVAR in (19) becomes

$$(22) \quad \begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix} = \begin{pmatrix} \alpha_c \\ \alpha_y \\ \alpha_w \\ \alpha_R \end{pmatrix} [c_{t-1} - \beta_y y_{t-1} - \beta_w w_{t-1} - \beta_R R_{t-1}] + \Gamma_1 \begin{pmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \\ \Delta w_{t-1} \\ \Delta R_{t-1} \end{pmatrix}$$
$$+\gamma t + \vartheta + \Phi D_t + \epsilon_t.$$

[10] Hall (1978) worked in levels rather than in logarithms of the variables in his regressions.

It follows that consumption is equilibrium correcting when $-1 < \alpha_c < 0$. However, income, wealth and the real after tax interest rate may also be equilibrium correcting if the corresponding value of alpha is positive and less than one. If $\alpha_y = \alpha_w = \alpha_R = 0$, on the other hand, income, wealth and the real after tax interest rate are all weakly exogenous with respect to β and the conditional Keynesian consumption function from (22) becomes

$$
\begin{aligned}
\Delta c_t &= \alpha_c[c_{t-1} - \beta_y y_{t-1} - \beta_w w_{t-1} - \beta_R R_{t-1}] + \omega_y \Delta y_t + \omega_w \Delta w_t + \omega_R \Delta R_t \\
&\quad + \tilde{\gamma}_{1,11} \Delta c_{t-1} + \tilde{\gamma}_{1,12} \Delta y_{t-1} + \tilde{\gamma}_{1,13} \Delta w_{t-1} + \tilde{\gamma}_{1,14} \Delta R_{t-1} \\
&\quad + \tilde{\gamma}_c t + \tilde{\vartheta}_c + \tilde{\Phi}_c D_t + \tilde{\epsilon}_{ct},
\end{aligned} \tag{23}
$$

where the inclusion of the contemporaneous variables, Δy_t, Δw_t and ΔR_t, follows from the properties of the multivariate normal error distribution and where the coefficients are linear functions of the coefficients in (22) and the parameters from the multivariate normal error distribution, see for instance Johansen (1995, p. 122).

We have seen that cointegration in (19) represents the common ground between the consumption Euler equation approach and the Keynesian consumption function approach and that the theoretical predictions from the two approaches put different restrictions with respect to weak exogeneity on consumption and income. In the empirical analysis, we shall therefore consider hypotheses of cointegration and equilibrium correction as restrictions on $\Pi = \alpha\beta'$, both before and after the financial crisis, in order to discriminate between the two approaches. Because CVAR models considering conditional expectations of future consumption and income may corroborate parameters stemming from the class of Euler equations, we shall also examine the empirical relevance of such models within the context of Johansen and Swensen (1999, 2004, 2008). Having established a nesting CVAR in theory, we now turn to the data underlying the empirical analysis.

3 Data

As previously mentioned, Jansen (2013) analyses cointegration between consumption, income and wealth conditioning on the real after tax interest rate for the sample period 1970q3 to 2008q2. For comparison reasons, we maintain the data set from that study *as is* and extend it by using quarterly growth rates from the final national accounts for the period 2008q3$-$2016q4, keeping 2008q2 fixed. As such, we follow both Brodin and Nymoen (1992), Jansen (2013) and Eitrheim et *al.* (2002) and work with non-per capita consumption, income and wealth in the empirical analysis. We note that (22) need not be specified in per capita terms by the population, N_t, because $C_t/N_t = (Y_t/N_t)^{(1-\beta_w)} \cdot (W_t/N_t)^{\beta_w}$ is equivalent to $c_t = (1-\beta_w)y_t + \beta_w w_t$ in the case of homogeneity between consumption, income and wealth. As we shall see, the homogeneity restriction is indeed supported by the data.[11]

Because the capital markets in Norway were heavily regulated during the 1970s and early 1980s, which likely prevented many consumers from acting freely in accordance with a consumption Euler equation, we choose 1984q1 as the starting point of our sample period. However, due to lags in the CVAR, the sample period for estimation purposes includes data points from 1982q3 to 2016q4. The sample period is thus consistent with the period of liberalisation of what was believed to be the most binding regulations of credit for households, namely the bond market which was deregulated in several steps between 1982 and 1985 to allow for competition among banks and other lending institutions in the household market. We also choose 2008q4 as the starting point of the financial crisis. Albeit the bankruptcy of Lehman Brothers took place the 15th of September 2008, we believe that the main effects on the Norwegian economy, and hence on the households' consumer behaviour, emerged in the fourth quarter of 2008 onwards.[12]

The consumption variable is defined as real consumption excluding expenditures on

[11]Erlandsen and Nymoen (2008), on the other hand, express their consumption function in per capita terms, but they emphasise that the results obtained do not depend in any substantive way on the per capita formulation.

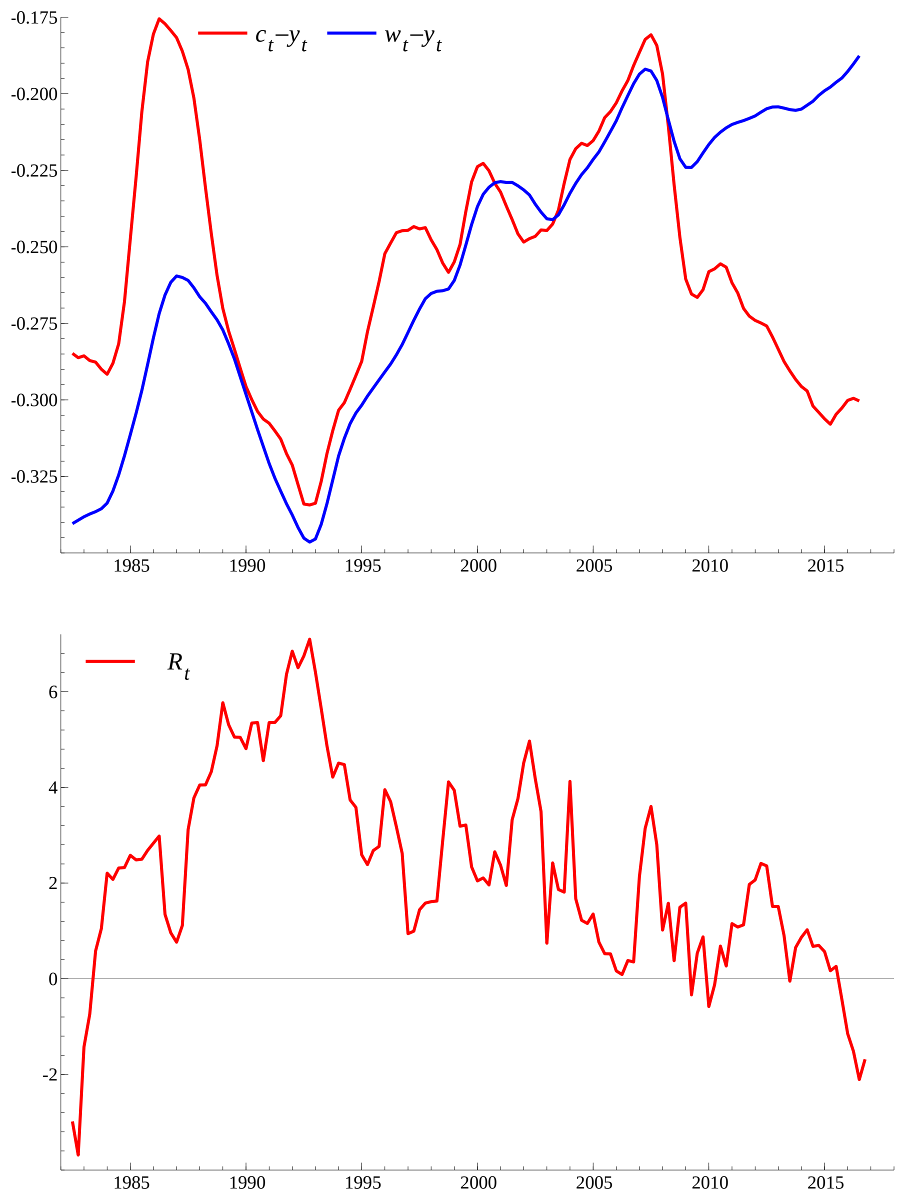
[12]We have checked that choosing 2008q3 as the starting point of the financial crisis does not alter the empirical findings reported below.

health services and housing. Expenditures on health services are excluded from the consumption variable as almost all of these are refunded by the government. Likewise, the imputed housing consumption is closely related to the imputed value of housing income by construction in the national accounts. Thus it does not make sense to include this component in the consumption variable when one purpose of our study is to estimate the MPC. Since we want to test the theory implications of the permanent income hypothesis, it could be argued that also durable goods other than housing should be excluded from the consumption variable under study, see for instance Deaton (1992, pp. 99-103). However, data inspection reveals that the ratio between consumption of durables and our consumption variable fluctuates around a constant level, which suggests long-run constancy. Taking logarithms means that the difference between the two consumption variables will be captured adequately by the intercept term in the consumption models under study.

The income variable is real disposable income excluding equity income. The latter is left out because of episodes where tax increases on equity incomes were announced for the coming year leading to substantial tax motivated fluctuations in this income component, bearing in mind that equity incomes are likely to be less motivating for consumption than other incomes. Likewise, the wealth variable is measured in real terms net of household debt and thus consists of the value of housing as well as total net financial wealth. These entities differ widely in terms of liquidity and availability for the purpose of consumption of goods and services. We have nonetheless maintained the aggregated wealth measure in the sequel. Finally, the real after tax interest rate is defined as the average nominal interest rates on bank loans faced by households net of marginal income tax and adjusted for inflation. In Appendix 1, we give more precise definitions of all the variables entering the empirical models in Sections 4 and 5.

Figure 1 shows the consumption to income and the wealth to income ratios together with the real after tax interest rate for the sample period $1982q3-2016q4$. We observe a strong co-movement between the two ratios in the sample period before the financial crisis

Figure 1: The consumption to income ($c_t - y_t$) ratio, the wealth to income ($w_t - y_t$) ratio and the real after tax interest rate (R_t)



Notes: Sample period: 1982q3$-$2016q4. Upper panel shows moving averages of the two ratios in logarithms, with one quarter lag and two quarters lead. Mean and range of the logarithms of wealth to income are matched to mean and range of the logarithms of consumption to income. Lower panel shows the real after tax interest rate measured in per cent per annum.

hit the Norwegian economy and this is *prima facie* evidence for cointegration between the three variables involved. However, a break in the cointegration relationship seems evident in the subsequent period as the two ratios then diverge and move in opposite directions. The real after tax interest rate for its part reached a historical high level in the early 1990s in the wake of the huge boom in consumption during the second half of the 1980s. Since then the real after tax interest rate has shown a downward trend and reached negative levels as in the early 1980s at the end of the sample period. While unit root tests suggest that the logarithms of consumption, income and wealth are all $I(1)$ variables, the tests are somewhat ambiguous with respect to the real after tax interest rate being either an $I(0)$ or $I(1)$ variable.[13] These features of the data are the premises for the cointegration analysis, which we now turn to.

4 Cointegration analysis[14]

In this section, we first carry out a multivariate cointegration analysis for the sample period prior to the financial crisis using (19) as the underlying model. Then, we conduct a similar cointegration analysis on the extended sample period ending in 2016q4 with a structural break around the financial crisis in 2008, applying the models and methods in Johansen *et al.* (2000).

4.1 The sample period prior to the financial crisis

We shall follow common practice and adopt the trace test for cointegration to determine the rank order of $\Pi = \alpha\beta'$, see e.g. Johansen (1995, p. 167), whereby the linear trend is restricted to lie within the cointegrating space and the parameters ϑ and Φ are kept unrestricted in

[13]Results from standard Augmented Dickey Fuller tests are available from the authors upon request. Based on unit root tests, Jansen (2013) suggests that the real after tax interest rate is non-stationary, while Anundsen and Jansen (2013) assume that the real after tax interest rate is stationary over the period from the mid 1980s to the end of 2008.

[14]The econometric modelling in this section is carried out with PcGive 14, see Doornik and Hendry (2013).

Table 1: Trace test results for cointegration[1]

Eigenvalue: λ_i	H_0	H_A	λ_{trace}
0.359	$r = 0$	$r \geq 1$	84.88 [0.000]
0.209	$r \leq 1$	$r \geq 2$	40.84 [0.078]
0.102	$r \leq 2$	$r \geq 3$	17.62 [0.377]
0.068	$r \leq 3$	$r = 4$	7.01 [0.354]

Diagnostics[2]	Test statistic	Value[p-value]
Vector autocorrelation 1-5 test:	F(80,187)	1.23 [0.253]
Vector normality test:	$\chi^2(8)$	6.95 [0.542]
Vector heteroscedasticity test:	F(212,170)	1.08 [0.307]

Sample period: 1982q3−2008q3. [1] See Johansen (1995), VAR of order 6, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: trend (restricted), constant (unrestricted) and centered seasonal dummies (unrestricted), r denotes the rank order of $\Pi = \alpha\beta'$ and λ_{trace} is the trace statistic with p-value in square brackets, as reported in PcGive 14. [2] See Doornik and Hendry (2013, p. 172).

(19). Hence, our underlying CVAR may be rewritten as

$$(24) \qquad \Delta X_t = \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \vartheta + \Phi D_t + \epsilon_t.$$

As a guidance for choosing the optimal lag length of the underlying VAR, we rely on the Akaike's information criterion (AIC), likelihood ratio tests of sequential model reduction and diagnostic tests of the residuals. According to both the AIC and the series of model reduction tests, the VAR in our case should include six lags, and not five lags as in Jansen (2013), as the premise for the cointegration analysis.[15] We also note that a shorter lag length than $k = 6$ produces significant departures from white noise residuals, especially in the equation for consumption, according to the diagnostic tests.

Table 1 displays results from the trace tests for cointegration and the diagnostic tests of the selected sixth order VAR. The model appears to be well-specified. The trace tests support the hypotheses of one and two cointegrating vector(s) between c_t, y_t, w_t and R_t at the 5 and 10 per cent significance level, respectively. We shall therefore consider both

[15] Results from the AIC and the model reduction tests are available from the authors upon request.

cases when testing restrictions on $\Pi = \alpha\beta'$ to discriminate between the consumption Euler equation and the Keynesian consumption function.

Assuming $r = 2$, we remember from Subsection 2.3 that the restriction $\alpha_{c1} = \alpha_{c2}$, which must be satisfied if a consumption Euler equation is true, can be tested empirically once the two cointegrating vectors are exactly identified. Hence, after imposing $\beta_{c1} = 1$ and $\beta_{w1} = 0$ in the first row of β' and $\beta_{c2} = -1$ and $\beta_{y2} = 1$ in the second row of β' to achieve exact identification, we can compare the log likelihood value of the CVAR with and without the restriction $\alpha_{c1} = \alpha_{c2}$ imposed. The associated likelihood ratio test statistic, $\chi^2(1) = 12.81$ with a p-value of 0.0003, points to strong rejection of the restriction. Thus consumption *is* equilibrium correcting in some way. We may therefore already at this stage of the analysis reject a consumption Euler equation and continue testing restrictions on $\Pi = \alpha\beta'$ under the assumption that $r = 1$ since the trace tests also support such an order of the rank.

Table 2 summarises main likelihood ratio tests of restrictions conditioning on the rank being unity. We see that the hypotheses of homogeneity between consumption, income and wealth and the irrelevance of the trend variable are not rejected, neither jointly (p-value = 0.42) nor individually (p-values = 0.19 and 0.22). So is the joint hypothesis $\beta_y = 1$, $\beta_w = 0$ and $\gamma = 0$ (p-value = 0.34), the joint hypothesis $\beta_y = 1$ and $\beta_w = 0$ (p-value = 0.39) and the individual hypothesis $\beta_w = 0$ (p-value = 0.60). Moreover, a likelihood ratio test, $\chi^2(1) = 1.6$ and p-value = 0.21, supports reduction from Model (*ii*) to Model (*iii*) and finding homogeneity in this case is compatible with one of the predictions of the consumption Euler equation. However, the estimates of β_R and α change considerably when imposing homogeneity between consumption and income only. For these reasons, and the fact that the t-value of the estimate of β_w in Model (*ii*) is around 2 in magnitude, we continue focusing on the cointegrating vector which also includes the wealth variable. The estimated adjustment coefficients, except from the estimate of α_y (p-value = 0.11), are all highly significant in Model (*iv*). Accordingly, consumption, and not income, equilibrium

Table 2: Likelihood ratio test results for restrictions on $\Pi = \alpha\beta'$[1]

Model (i): $\beta_c = 1$

$$c_t - \underset{(0.29)}{1.26} y_t - \underset{(0.05)}{0.06} w_t + \underset{(0.44)}{2.63} R_t + \underset{(0.0017)}{0.0022} t$$

$$\hat{\alpha}_c = -\underset{(0.07)}{0.26},\ \hat{\alpha}_y = -\underset{(0.05)}{0.09},\ \hat{\alpha}_w = -\underset{(0.09)}{0.29},\ \hat{\alpha}_R = -\underset{(0.03)}{0.11}$$

$\log L = 1170.04$

Model (ii): $\beta_c = 1$, $\beta_y + \beta_w = 1$, $\gamma = 0$

$$c_t - 0.94 y_t - \underset{(0.03)}{0.06} w_t + \underset{(0.47)}{3.07} R_t$$

$$\hat{\alpha}_c = -\underset{(0.06)}{0.21},\ \hat{\alpha}_y = -\underset{(0.05)}{0.09},\ \hat{\alpha}_w = -\underset{(0.08)}{0.25},\ \hat{\alpha}_R = -\underset{(0.02)}{0.10}$$

$\log L = 1169.17$

$\chi^2(2) = 1.75[0.42]$[2], $\chi^2(1) = 1.75[0.19]$[3], $\chi^2(1) = 1.48[0.22]$[4]

Model (iii): $\beta_c = 1$, $\beta_y = 1$, $\beta_w = 0$, $\gamma = 0$

$$c_t - y_t + \underset{(0.56)}{4.3} R_t$$

$$\hat{\alpha}_c = -\underset{(0.05)}{0.14},\ \hat{\alpha}_y = -\underset{(0.04)}{0.05},\ \hat{\alpha}_w = -\underset{(0.06)}{0.20},\ \hat{\alpha}_R = -\underset{(0.02)}{0.08}$$

$\log L = 1168.37$

$\chi^2(3) = 3.35[0.34]$[5], $\chi^2(2) = 1.91[0.39]$[6], $\chi^2(1) = 0.27[0.60]$[7]

Model (iv): $\beta_c = 1$, $\beta_y + \beta_w = 1$, $\gamma = 0$, $\alpha_y = 0$

$$c_t - 0.96 y_t - \underset{(0.03)}{0.04} w_t + \underset{(0.52)}{3.30} R_t,$$

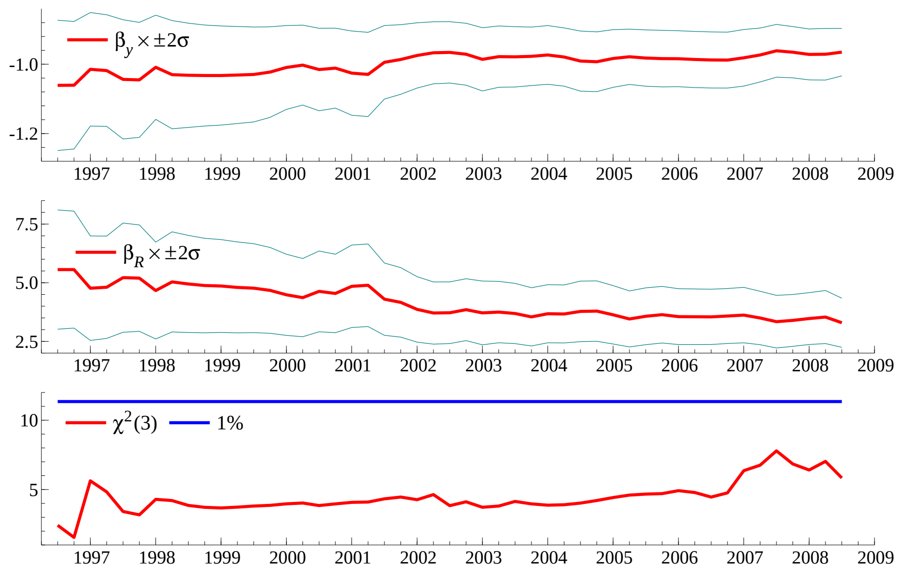
$$\hat{\alpha}_c = -\underset{(0.06)}{0.19},\ \hat{\alpha}_w = -\underset{(0.07)}{0.27},\ \hat{\alpha}_R = -\underset{(0.02)}{0.09}$$

$\log L = 1167.13$

$\chi^2(3) = 5.84[0.12]$[8], $\chi^2(1) = 2.57[0.11]$[9]

Sample period: $1982q3-2008q3$. [1] See Johansen (1995), VAR of order 6, $r = 1$, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: trend (restricted), constant (unrestricted) and centered seasonal dummies (unrestricted), standard errors in parenthesis and p-values in square brackets. [2] $\beta_y + \beta_w = 1$, $\gamma = 0$. [3] $\beta_y + \beta_w = 1$. [4] $\gamma = 0$. [5] $\beta_y = 1$, $\beta_w = 0$, $\gamma = 0$. [6] $\beta_y = 1$, $\beta_w = 0$. [7] $\beta_w = 0$. [8] $\beta_y + \beta_w = 1$, $\gamma = 0$, $\alpha_y = 0$. [9] $\alpha_y = 0$.

Figure 2: Recursive estimates of restricted long-run coefficients



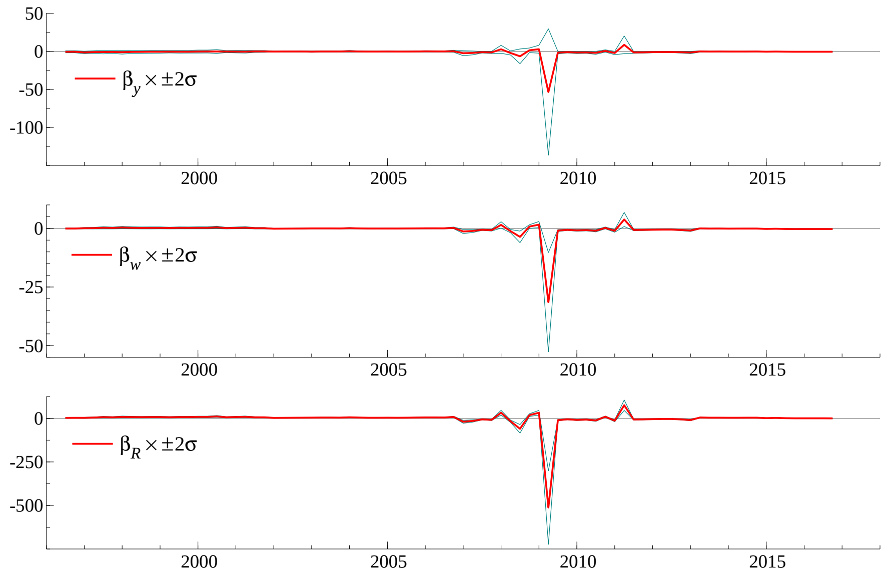
Notes: Sample period: $1982q3-2008q3$.

corrects in the CVAR, which clearly contradicts an important prediction of the consumption Euler equation. Imposing the hypotheses of homogeneity, irrelevance of the trend variable and income being weakly exogenous, yields the following restricted long-run relationship

$$\widehat{eqcm}_{1,t} = c_t - 0.96y_t - 0.04w_t + 3.30R_t. \tag{25}$$

Figure 2 shows recursive estimates of the long-run coefficients in (25). It is evident that the coefficients for income, and hence for wealth, as well as for the real after tax interest rate are reasonably stable. Also, the recursive likelihood ratio tests support the joint hypothesis of $\beta_y + \beta_w = 1$, $\gamma = 0$ and $\alpha_y = 0$. A comparison with equation (2) in Jansen (2013) shows that the estimated income elasticity in (25) is somewhat higher, the estimated wealth elasticity somewhat lower and, maybe more importantly, that the estimated real after tax interest rate semi-elasticity is more than four times as high. Apart from different sample periods, a possible explanation may be that Jansen (2013), as previously noted, considers partial CVAR models in which the real after tax interest rate is conditioned upon at the

Figure 3: Recursive estimates of unrestricted long-run coefficients



Notes: Sample period: 1982q3$-$2016q4.

outset. Because our findings suggest that this variable is far from being weakly exogenous, $\chi^2(1) = 15.35$ and p-value $= 0.0001$, there may be considerable loss of information of not taking into account that property in the cointegration analysis.

It follows from all the findings above that the data support a Keynesian type consumption function over the sample period prior to the financial crisis.

4.2 The extended sample period

When extending the sample period with 33 additional quarters, up to and including 2016q4, we continue to build on (24) with $k = 6$ as the underlying model.[16] Before conducting a re-analysis of restrictions on $\Pi = \alpha\beta'$, we shall examine the stability properties of the unrestricted counterpart of (25) when the sample period is extended with the additional data points. Figure 3 shows recursive estimates of the long-run coefficients, assuming the rank to be unity, over the extended sample period. Unlike the estimated coefficients over the sample

[16] Again, the AIC and the series of model reduction tests support the choice of six lags in the VAR.

period from the mid 1980s to the end of 2008, we clearly see that the estimated coefficients are unstable and reveal a significant structural break in the long-run relationship around the financial crisis. A possible explanation of the structural break is that the underlying VAR suffers from omitted variables necessary to explain a changing consumer behaviour after the financial crisis hit the Norwegian economy.

Following Johansen *et al.* (2000), a structural break in the long-run relationship can be captured by a model which takes into account the possibility of separate trends in the two periods $1, \ldots, T_1$ and $T_1+1, \ldots, T$. The idea is to allow for two VAR models where the k first observations are conditioned upon, but where the parameters of the stochastic components are the same for both models and where the parameters of the deterministic components may differ corresponding to a broken trend. Formally, let $T_0 = 0$ and $T_2 = T$. If $ID_{j,t} = 1$ for $t = T_{j-1}$ and $ID_{j,t} = 0$ else so that $ID_{j,t-i}$ is the indicator for the ith observation in the jth period, $j = 1, 2$, it follows that $SD_{j,t} = \Sigma_{i=k+1}^{T_j - T_{j-1}} ID_{j,t-i} = 1$ for $t = T_{j-1} + k + 1, \ldots, T_j$ and $SD_{j,t} = 0$ else. The CVAR in (24) is then reformulated for $t = k + 1, \ldots, T$ as

$$
\begin{aligned}
(26) \qquad \Delta X_t \;=\; & \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ tSD_t \end{pmatrix} + \mu SD_t + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} \\
& + \Phi D_t + \kappa_{2,1} ID_{2,t-1} + \ldots + \kappa_{2,k} ID_{2,t-k} + \epsilon_t,
\end{aligned}
$$

where $SD_t = (SD_{1,t}, SD_{2,t})'$, $\gamma = (\gamma_1', \gamma_2')'$ and $\mu = (\mu_1', \mu_2')'$. In our case we assume, referring to Figure 1 and the discussion in Section 3, that the break occurs in 2008 $q4$. This means that we augment the sixth order VAR by defining $SD_{1,t}$ as a step dummy which is unity in the period 1982q3−2008q3, $SD_{2,t}$ as a step dummy which is unity in the period 2010q2−2016q4 and $ID_{2,t}$ as impulse indicators which are unity for $t =$ 2008q4,...,2010q1, otherwise zero.

We can then repeat the cointegration analysis with the augmented VAR, letting SD_t and $ID_{2,t}$ enter the VAR unrestrictedly, whereas tSD_t is restricted to lie in the cointegration space. The trace test results for cointegration are shown in Table 3 together with the

Table 3: Trace test results for cointegration with a structural break[1]

Eigenvalue: λ_i	H_0	H_A	λ_{trace}
0.287	$r = 0$	$r \geq 1$	101.21 [0.001]
0.188	$r \leq 1$	$r \geq 2$	56.49 [0.050]
0.129	$r \leq 2$	$r \geq 3$	28.96 [0.207]
0.078	$r \leq 3$	$r = 4$	10.74 [0.372]

Diagnostics[2]	Test statistic	Value[p-value]
Vector autocorrelation 1-5 test:	F(80,286)	1.37 [0.034]
Vector normality test:	$\chi^2(8)$	9.02 [0.341]
Vector heteroscedasticity test:	F(224,266)	1.03 [0.407]

Sample period: 1982q3$-$2016q4. [1] See Johansen *et al.* (2000), VAR of order 6, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: tSD_t (restricted), SD_t (unrestricted), ID_{2t} (unrestricted) and centered seasonal dummies (unrestricted), r denotes the rank order of $\Pi = \alpha\beta'$ and λ_{trace} is the trace statistic with p-value in square brackets, which are calculated by means of the estimated response surface in Johansen *et al.* (2000, Table 4). [2] See Doornik and Hendry (2013, p. 172).

diagnostic tests of the underlying sixth order VAR. Again, the estimated model has no serious departures from white noise residuals, albeit the test for autocorrelation until the fifth order is a borderline case at the 5 per cent significance level. The trace tests now support marginally the hypothesis of two cointegrating vectors between c_t, y_t, w_t and R_t at the 5 per cent significance level. We shall therefore continue to consider two cases, $r = 2$ and $r = 1$, when testing restrictions on $\Pi = \alpha\beta'$ to discriminate between the consumption Euler equation and the Keynesian consumption function over the extended sample period.

Starting with $r = 2$, we find, once the two cointegrating vectors are exactly identified in the same fashion as described above, that the restriction $\alpha_{c1} = \alpha_{c2}$ still is strongly rejected by the likelihood ratio statistic, which now becomes $\chi^2(1) = 10.65$ with a p-value of 0.001. We again conclude that the data overwhelmingly reject a consumption Euler equation.

Table 4 reports main likelihood ratio tests of restrictions on $\Pi = \alpha\beta'$ with a structural break around the financial crisis assuming $r = 1$. The hypothesis of homogeneity between consumption, income and wealth is, as before, accepted by the data in Model (*ii*). Note that the trend variable for the first period is *not* excluded from the model as the estimate of γ_1 is a borderline case at the 10 per cent significance level (p-value $= 0.103$). A likelihood ratio

Table 4: Likelihood ratio test results for restrictions on $\Pi = \alpha\beta'$ with a structural break[1]

Model (i): $\beta_c = 1$

$$c_{t-1} - \underset{(0.23)}{1.06} y_{t-1} - \underset{(0.039)}{0.16} w_{t-1} + \underset{(0.32)}{1.72} R_{t-1} + \underset{(0.0013)}{0.0023} tSD_{1,t} + \underset{(0.0020)}{0.0066} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.09)}{0.31},\ \hat{\alpha}_y = \underset{(0.07)}{0.003},\ \hat{\alpha}_w = -\underset{(0.11)}{0.27},\ \hat{\alpha}_R = -\underset{(0.03)}{0.15}$$

$\log L = 1558.35$

Model (ii): $\beta_c = 1,\ \beta_y + \beta_w = 1$

$$c_{t-1} - 0.84 y_{t-1} - \underset{(0.04)}{0.16} w_{t-1} + \underset{(0.35)}{1.97} R_{t-1} + \underset{(0.00028)}{0.00089} tSD_{1,t} + \underset{(0.001)}{0.0051} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.08)}{0.26},\ \hat{\alpha}_y = -\underset{(0.07)}{0.008},\ \hat{\alpha}_w = -\underset{(0.10)}{0.24},\ \hat{\alpha}_R = -\underset{(0.03)}{0.14}$$

$\log L = 1557.71$

$\chi^2(1) = 1.28[0.26]^2$

Model (iii): $\beta_c = 1,\ \beta_y = 1,\ \beta_w = 0$

$$c_{t-1} - y_{t-1} + \underset{(0.63)}{4.28} R_{t-1} + \underset{(0.00035)}{0.00026} tSD_{1,t} + \underset{(0.0021)}{0.0074} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.04)}{0.10},\ \hat{\alpha}_y = -\underset{(0.03)}{0.026},\ \hat{\alpha}_w = -\underset{(0.05)}{0.15},\ \hat{\alpha}_R = -\underset{(0.01)}{0.07}$$

$\log L = 1556.74$

$\chi^2(2) = 3.21[0.20]^3,\ \chi^2(1) = 2.16[0.14]^4$

Model (iv): $\beta_c = 1,\ \beta_y + \beta_w = 1,\ \alpha_y = 0$

$$c_{t-1} - 0.83 y_{t-1} - \underset{(0.04)}{0.17} w_{t-1} + \underset{(0.35)}{1.93} R_{t-1} + \underset{(0.00028)}{0.00091} tSD_{1,t} + \underset{(0.00099)}{0.0050} tSD_{2,t}$$

$$\hat{\alpha}_c = -\underset{(0.08)}{0.26},\ \hat{\alpha}_w = -\underset{(0.10)}{0.24},\ \hat{\alpha}_R = -\underset{(0.03)}{0.15}$$

$\log L = 1557.70$

$\chi^2(2) = 1.28[0.53]^5,\ \chi^2(1) = 0.002[0.97]^6$

Sample period: $1982q3-2016q4$. [1] See Johansen *et al.* (2000), VAR of order 6 with a structural break in $2008q4$, $r = 1$, modelled variables: c_t, y_t, w_t and R_t, deterministic variables: $tSD_{1,t}$ and $tSD_{2,t}$ (restricted), $SD_{1,t}$ and $SD_{2,t}$ (unrestricted), $ID_{2,t}$ (unrestricted) and centered seasonal dummies (unrestricted), standard errors in parenthesis, p-values in square brackets. [2] $\beta_y + \beta_w = 1$. [3] $\beta_y = 1$, $\beta_w = 0$. [4] $\beta_w = 0$. [5] $\beta_y + \beta_w = 1$, $\alpha_y = 0$. [6] $\alpha_y = 0$.

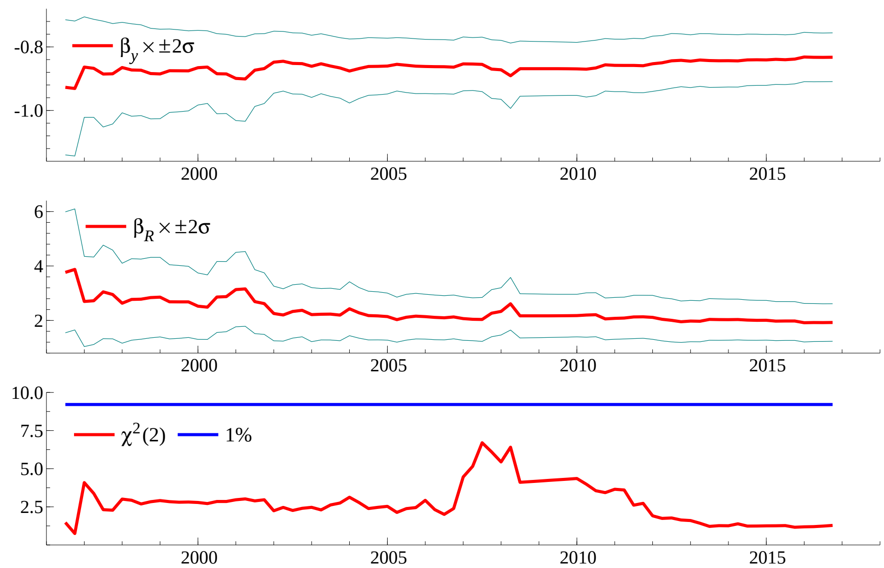
test, $\chi^2(1) = 1.94$ and p-value $= 0.16$, again supports reduction from Model (*ii*) to Model (*iii*) in which homogeneity between consumption and income and exclusion of the wealth variable are imposed.[17] However, the p-value of the individual hypothesis $\beta_w = 0$ drops from 0.60 for the sample period prior to the financial crisis to 0.14 for the full sample period and the associated t-value is now as high as 4 in magnitude in Model (*ii*). In addition, the estimates of β_R and α change significantly, as before, when imposing homogeneity between consumption and income only. We therefore continue to keep the wealth variable in the cointegrating vector and find that the hypothesis $\alpha_y = 0$ is strongly supported by the data (p-value $= 0.97$) in Model (*iv*). When imposing homogeneity between consumption, income and wealth and weak exogeneity of income, the restricted long-run relationship becomes

$$\widehat{eqcm}_{2,t} = c_{t-1} - 0.83y_{t-1} - 0.17w_{t-1} + 1.93R_{t-1} + 0.00091tSD_{1,t} + 0.0050tSD_{2,t}. \tag{27}$$

Figure 4 shows that the estimated coefficients of y_t, and hence also of w_t, as well as of R_t, are stable before, during and after the financial crisis once the structural break is allowed for. Also, the recursive likelihood ratio tests support the joint hypothesis of $\beta_y + \beta_w = 1$ and $\alpha_y = 0$. A notable difference between (25) and (27) is that the estimated real after tax interest rate semi-elasticity has dropped to almost a half in magnitude. And, compared to Jansen (2013), the estimated elasticities of income and wealth are almost perfectly reproduced on the sample period ending in 2016q4. A possible interpretation of the fact that the semi-elasticity of the real after tax interest rate has decreased and the elasticity of wealth has increased after the financial crisis is the following: After the financial crisis households have faced increased credit constraints by *inter alia* lending criteria based on payment-to-income ratios due to increased credit risk in the economy. Households have thus not been able to borrow at the observed lending interest rates as easily as before the financial crisis because of tightening of the credit practises. As a consequence, households credit worthiness, as

[17] The likelihood ratio test statistic becomes $\chi^2(1) = 2.47$ (p-value $= 0.11$) when also imposing a zero restriction on α_y, whose estimate is strongly *in*significant in both Model (*ii*) and Model (*iii*).

Figure 4: Recursive estimates of restricted long-run coefficients



Notes: Sample period: $1982q3-2016q4$.

measured by total wealth, has become increasingly important for mortgage and other loan security after the financial crisis, and thus also for the ability to borrow for consumption purposes.

We also find that the deterministic trend in (27) is significantly shifting equilibrium consumption downwards both before and after the financial crisis. However, the shift is much larger after $2008q4$, with a factor of 5.7 according to model (ii) $-$ which corresponds to the modelling assumptions in Section 5 $-$ and a factor of 5.5 according to model (iv). A possible interpretation may be that the broken trend reflects increased uncertainty and thus increased precautionary savings in the wake of the financial crisis. The fact that the households' saving ratio increased from nearly 4 per cent in 2008 to more than 10 per cent in 2015 supports this conjecture.

To facilitate a comparison of the magnitude of the MPC implied by Model A1 in Table 4 in Jansen (2013), we perform a reduced rank regression for a partial model over the sample period $1982q3$ to $2016q4$ following the modelling strategy of equation (10) in Harbo *et al.* (1998). Since the hypothesis of weak exogeneity of income with respect to the long-run parameters is supported by the data, we can without loss of information condition on this variable when estimating a partial CVAR for consumption, wealth and the real after tax interest rate. Our point of departure is therefore the partial model written in vector form as

$$\Delta X_t^* = \Theta_D D_t + \sum_{j=0}^{5} \omega_{Z,j} \Delta Z_{t-j} + \sum_{j=1}^{5} \Theta_{X^*,j} \Delta X_{t-j}^* + \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ tSD_t \end{pmatrix} + \varepsilon_t, \tag{28}$$

where $X_t^* = (c_t, w_t, R_t)'$, $Z_t = y_t$, $X_t = (c_t, y_t, w_t, R_t)'$ and D_t includes the centered seasonal dummies and all the dummies for the structural break around the financial crisis. We estimate (28) by constrained full information maximum likelihood (CFIML) whereby the rank is restricted to unity and the hypothesis of homogeneity between consumption, income and wealth is imposed in accordance with the evidence above. The estimated consumption equation on the general form is stated below together with standard errors of estimated coefficients (in parenthesis), standard error of equation ($\hat{\sigma}$) and system diagnostics for autocorrelation, normality and heteroscedasticity (p-values in square brackets), see Doornik and Hendry (2013, p. 172).[18]

Interestingly, the consumption equation in (29) below implies a first year MPC of around 25 per cent, which is quite close to 30 per cent implied by Model A1 in Jansen (2013).[19] These findings are in line with the argument in Doornik and Hendry (1997) that the main source of forecast failure is deterministic shifts in equilibrium means, e.g. the equilibrium saving ratio, and not shifts in the derivative coefficients, e.g. the marginal propensity to consume, that are of primary interest for policy analysis. Our estimate is also consistent with the aggregate MPC of around 20 per cent predicted by Carroll *et al.* (2017)

[18] The estimated equations for Δw_t and ΔR_t are reported in Appendix 2.

[19] We note that simplifying (28), general-to-specific, by deleting non-significant terms, starting with the last lag of the real interest rate followed by the last lag of wealth, income and consumption, does not change the estimated magnitude of the MPC.

in a model with heterogeneous agents.

$$
\begin{aligned}
(29) \qquad \Delta\widehat{c}_t \;=\; & -\underset{(0.13)}{0.47}\Delta c_{t-1} - \underset{(0.13)}{0.19}\Delta c_{t-2} + \underset{(0.12)}{0.06}\Delta c_{t-3} + \underset{(0.11)}{0.52}\Delta c_{t-4} + \underset{(0.10)}{0.29}\Delta c_{t-5} \\
& + \underset{(0.12)}{0.12}\Delta y_t - \underset{(0.17)}{0.13}\Delta y_{t-1} - \underset{(0.18)}{0.14}\Delta y_{t-2} + \underset{(0.18)}{0.22}\Delta y_{t-3} + \underset{(0.17)}{0.38}\Delta y_{t-4} \\
& + \underset{(0.13)}{0.20}\Delta y_{t-5} + \underset{(0.08)}{0.26}\Delta w_{t-1} + \underset{(0.09)}{0.07}\Delta w_{t-2} - \underset{(0.08)}{0.06}\Delta w_{t-3} - \underset{(0.08)}{0.18}\Delta w_{t-4} \\
& - \underset{(0.08)}{0.10}\Delta w_{t-5} + \underset{(0.28)}{0.49}\Delta R_{t-1} + \underset{(0.25)}{0.30}\Delta R_{t-2} - \underset{(0.26)}{0.04}\Delta R_{t-3} - \underset{(0.23)}{0.36}\Delta R_{t-4} \\
& + \underset{(0.24)}{0.03}\Delta R_{t-5} - \underset{(0.10)}{0.26}c_{t-1} + \underset{(0.08)}{0.22}y_{t-1} + \underset{(0.02)}{0.04}w_{t-1} - \underset{(0.16)}{0.51}R_{t-1} \\
& - \underset{(0.0001)}{0.0002}tSD_{1,t} - \underset{(0.0005)}{0.0013}tSD_{2,t} + terms\ involving\ dummies \\
& \hat{\sigma}_c = 0.019 \\
& CFIML, T = 132\ (1982q3 - 2016q4) \\
& Vector\ AR_{1-5}\text{: } F(45, 241) = 1.37[0.07] \\
& Vector\ NORM\text{: } \chi^2(6) = 10.26[0.11] \\
& Vector\ HET\text{: } F(348, 379) = 1.16[0.07]
\end{aligned}
$$

Based on the findings from testing restrictions on $\Pi = \alpha\beta'$ in the CVAR, both before and after the financial crisis, we conclude that the data support a Keynesian consumption function rather than a consumption Euler equation. Left unanswered, however, is whether conditional expectations of future consumption and income play a role in explaining the consumer behaviour.

5 Conditional expectations[20]

We recall from Subsection 2.3 that the conditional expectations of future consumption and income in (21) nests all the hypotheses in (15) with some rule of thumb consumers. Hence, we are motivated to examine the empirical relevance of CVAR models considering condi-

[20] The estimation and testing in this section are performed with the statistical package R, see http://www.r-project.org/.

tional expectations of future consumption and income within the context of Johansen and Swensen (1999, 2004, 2008), building on the findings in Section 4. That is, we let the rank order of the impact matrix be unity, but do not restrict income to be weakly exogenous to match the underlying assumptions of the methods used in this section. First, we outline the estimation procedure, paying particular attention to the conditional expectations restrictions on the stochastic part of the CVAR. Then, we estimate CVAR models with conditional expectations and examine whether data can corroborate parameters stemming from the class of Euler equations, both before and after the financial crisis. Specifically, we simplify the conditional expectations, general-to-specific, by deleting non-significant terms in the CVAR models, starting with the last lag. As such, we rely on a general-to-specific approach akin to the backward selection process common in linear models. We shall throughout the analysis simplify matters by specifying CVAR models in their *exact* form and not introduce a stochastic error term. As discussed in Boug *et al.* (2017), the numerical treatment of *inexact* models is complicated to handle using likelihood-based methods when a multivariate VAR is the underlying model.

5.1 Outline of the estimation procedure

Our main reference is still the CVAR in (24), which we repeat here for convenience.

$$\Delta X_t = \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ t \end{pmatrix} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \vartheta + \Phi D_t + \epsilon_t, \tag{30}$$

where $X_t = (c_t, y_t, w_t, R_t)'$ and D_t contains the centered seasonal dummies. The Euler equations involving expectations of future variables can be expressed as $c' E_t \Delta X_{t+1} = d' X_t$, which implies restrictions on the coefficients in (30). For instance, a bivariate system where the variables satisfy a martingale hypothesis can be written $(1,0)E_t(X_{1,t+1}, X_{2,t+1})' = (1,0)(X_{1,t}, X_{2,t})'$ or $(1,0)E_t\Delta(X_{1,t+1}, X_{2,t+1})' = 0$. Often it is convenient to have a more flexible specification of the form

$$c'E_t\Delta X_{t+1} - d'X_t + d'_{-1}\Delta X_t + \ldots + d'_{-k+1}\Delta X_{t-k+2} + \vartheta_0 + \gamma_0 t + \Phi_0 D_t = 0 \tag{31}$$

where c, d, d_{-1},...,d_{-k+1}, ϑ_0, γ_0 and Φ_0 are known matrices.

A flexible formulation arises by assuming that the $p\times q$ matrix c is known and allowing d, d_{-1},...,d_{-k+1}, ϑ_0, γ_0 and Φ_0 to be treated as matrices of unknown parameters. If they are allowed to vary freely (31) does not imply any constraints. It is only a statement that the conditional expectations of the next observation can be expressed by past and present values. The more general formulation can be exploited by testing whether any of the matrices d, $d_{-1}, \ldots, d_{-k+1}$, ϑ_0, γ_0 and Φ_0 in (31) are equal to zero or any given matrix. In other words, to investigate whether a simplification of the conditional expectations relation is possible.

The restrictions implied by (31) can be taken into account as follows: Leading (30) one period, the conditional expectations of ΔX_{t+1}, $E_t\Delta X_{t+1}$, given the present and previous values of X_t, can be expressed as

$$E_t\Delta X_{t+1} - \alpha\beta' X_t - \Gamma_1\Delta X_t - \ldots - \Gamma_{k-1}\Delta X_{t-(k-2)} - \vartheta - \alpha\gamma'(t+1) - \Phi D_{t+1} = 0 \tag{32}$$

The coefficients restrictions of the form (31) is thus equivalent to

$$\begin{gathered} c'\alpha\beta' = d', c'\Gamma_1 = -d'_{-1}, \ldots, c'\Gamma_{k-1} = -d'_{-k+1}, \\ c'\vartheta + c'\alpha\gamma'(t+1) + c'\Phi D_{t+1} = -\vartheta_0 - \gamma_0\, t - \Phi_0 D_t. \end{gathered} \tag{33}$$

These restrictions consist of two separate parts: One set of restrictions on the stochastic variables and another one on the deterministic variables. In the following, we concentrate on the stochastic part and leave the coefficients of the deterministic terms unrestricted.

Using the methods described in Johansen and Swensen (1999, 2004, 2008) the value of the concentrated likelihood $L_c(d, d_{-1}, \ldots, d_{-k+1}, \vartheta_0, \gamma_0, \Phi_0)$, where the restrictions given in

(31) are imposed, can be computed. Further maximization over $d, d_{-1}, \ldots, d_{-k+1}, \vartheta_0, \gamma_0$ and Φ_0 yields a value $\max L_c(d, d_{-1}, \ldots, d_{-k+1}, \vartheta_0, \gamma_0, \Phi_0)$ which is equal to the maximal value of the likelihood for (30), denoted as L_{max}. The likelihood ratio for a test of a particular hypothesis, for instance $d_{-k+1} = d^0_{-k+1}$, can then be found as

$$\frac{\max_{d,d_{-1},\ldots,d_{-k+1},\vartheta_0,\gamma_0,\Phi_0} L_c(d, d_{-1}, \ldots, d^0_{-k+1}, \vartheta_0, \gamma_0, \Phi_0)}{\max_{d,d_{-1},\ldots,d_{-k+2},d_{-k+1},\vartheta_0,\gamma_0,\Phi_0} L_c(d, d_{-1}, \ldots, d_{-k+1}, \vartheta_0, \gamma_0, \Phi_0)} = \frac{\max_{d,d_{-1},\ldots,d_{-k+2},\vartheta_0,\gamma_0,\Phi_0} L_c(d, d_{-1}, \ldots, d^0_{-k+1}, \vartheta_0, \gamma_0, \Phi_0)}{L_{max}}.$$

By considering the details of the methods described in Johansen and Swensen (1999, 2004, 2008) we can see that the maximization with respect to $d_{-1}, \ldots, d_{-k+2}, \vartheta_0, \gamma_0$ and Φ_0 can be performed by ordinary least squares (OLS) and reduced rank regression, while maximizing with respect to d must be carried out using numerical optimization. A more detailed explanation of the procedure can be found in Appendix 3.

5.2 Estimation with no break in trend

When considering conditional expectations of consumption in the next period, such that $c = (1, 0, 0, 0)'$, (30) takes the form

$$(34) \quad E_t\Delta c_{t+1} = \alpha_c(1, \beta_y, \beta_w, \beta_R, \gamma)\begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \\ t+1 \end{pmatrix} + (\gamma_{1,11}, \gamma_{1,12}, \gamma_{1,13}, \gamma_{1,14})\begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix}$$
$$+ \ldots + (\gamma_{5,11}, \gamma_{5,12}, \gamma_{5,13}, \gamma_{5,14})\begin{pmatrix} \Delta c_{t-4} \\ \Delta y_{t-4} \\ \Delta w_{t-4} \\ \Delta R_{t-4} \end{pmatrix} + c'\vartheta + c'\Phi D_{t+1},$$

Table 5: Likelihood ratio test results for simplifying restrictions[1] on (34)[2]

Model	Restrictions	$\log L_i$	$i-j$[3]	$-2\log\frac{L_j}{L_i}$	df	p-value
1	-	1170.04	-	-	-	
2	$\beta_y+\beta_w=1,\ \gamma=0$	1169.17	1-2	1.74	2	0.61
3	Model 2, $\gamma_{5,14}=0$	1169.15	2-3	0.04	1	0.84
4	Model 3, $\gamma_{5,13}=0$	1168.90	3-4	0.50	1	0.48
5	Model 4, $\gamma_{5,12}=0$	1161.55	4-5	14.70	1	0.0001
6	Model 4, $\gamma_{4,14}=0$	1167.79	4-6	2.22	1	0.14
7	Model 6, $\gamma_{4,13}=0$	1156.52	6-7	22.54	1	0.0000
8	Model 6, $\gamma_{3,14}=0$	1160.55	6-8	14.48	1	0.0001

Sample period: $1982q3-2008q3$. [1] See Johansen and Swensen (1999, 2004, 2008). [2] Model with no break in trend. [3] $i-j$ denotes the likelihood ratio test for the additional restriction(s) on model j compared to model i.

where the first row of the matrix Γ_j is denoted as $(\gamma_{j,11}, \gamma_{j,12}, \gamma_{j,13}, \gamma_{j,14})$ for $j = 1, \ldots, 5$. Table 5 reports likelihood ratio test results for simplifying restrictions on the coefficients of (34) with no break in trend over the sample period $1982q3-2008q3$.

When there are no restrictions on the coefficients of the differences, no restrictions on the coefficients of the dummies and the likelihood is maximized over β and γ, no restrictions are imposed, and the value of the maximum of the likelihood is the same as for (30). The log likelihood value in this case is displayed in the first line of Table 5. The second line shows the likelihood ratio test results when imposing the joint hypothesis of homogeneity in income and wealth and zero coefficient on the trend, cf. Model (*ii*) in Table 2. To simplify the model, general-to-specific, we test the significance of the estimated coefficients of the differences, starting with the last lag of the real interest rate. If the last lag of the real interest rate can be deleted from the model, we test the significance of the estimated coefficient of the last lag of wealth jointly with $\gamma_{5,14} = 0$ and continue similarly for income and consumption. This sequential way of testing estimated coefficients is then continued for the second last lag to the first lag until a specific model is established with no further significant simplifying restrictions. As seen from lines 3-8 of Table 5, we end up with Model 6 in which five lags of both consumption and income growth, four lags of wealth growth and three lags of growth in the real interest rate are retained. The estimated version of Model 6, with estimated standard errors in parenthesis, apart from the estimated coefficients of the dummies, is

$$
\begin{aligned}
(35) \qquad \widehat{E_t \Delta c_{t+1}} \;=\; & -0.23(1.0, -0.93, -0.07, 2.9) \begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \end{pmatrix} \\
& -\underset{(0.11)}{0.41}\Delta c_t - \underset{(0.11)}{0.15}\Delta c_{t-1} + \underset{(0.11)}{0.11}\Delta c_{t-2} + \underset{(0.11)}{0.38}\Delta c_{t-3} + \underset{(0.10)}{0.20}\Delta c_{t-4} \\
& -\underset{(0.14)}{0.43}\Delta y_t - \underset{(0.18)}{0.33}\Delta y_{t-1} + \underset{(0.11)}{0.20}\Delta y_{t-2} + \underset{(0.19)}{0.26}\Delta y_{t-3} + \underset{(0.15)}{0.14}\Delta y_{t-4} \\
& +\underset{(0.08)}{0.25}\Delta w_t + \underset{(0.09)}{0.11}\Delta w_{t-1} - \underset{(0.09)}{0.10}\Delta w_{t-2} - \underset{(0.09)}{0.29}\Delta w_{t-3} \\
& +\underset{(0.26)}{0.33}\Delta R_t + \underset{(0.26)}{0.54}\Delta R_{t-1} + \underset{(0.27)}{0.07}\Delta R_{t-2} + c'\hat{\vartheta}_0 + c'\hat{\Phi} D_{t+1}.
\end{aligned}
$$

As already pointed out, it is useful to consider the parameters as consisting of two parts: the first part, the parameters of interest, are the parameters stemming from the Euler equations described in Subsections 2.1 and 2.2 and the second part, the nuisance parameters, are the parameters necessary to ensure empirically well-specified models. There are several interesting consequences of (35). The first is a clear rejection of the hypothesis that log consumption is a martingale, $E_t[\Delta c_{t+1}] = 0$, which is a variant of the hypothesis of Hall (1978) that the level of consumption is a martingale. The result found in Jansen (2013) is therefore confirmed. The second is a comparison with the relation

$$
(36) \qquad E_t \Delta c_{t+1} = \phi + \sigma R_t,
$$

which, as outlined in Subsection 2.2, is derived using CRRA utility preferences. The implicit restrictions in (36) are clearly rejected since (35) cannot be reduced further. The third is evidence of some habit formation through lags of consumption growth in line with the Smets and Wouters (2003) model in (17). Finally, we may rewrite (34) as

$$
(37)\ \Delta c_t \quad = \quad \frac{-\alpha_c}{\gamma_{1,11}}(1,\beta_y,\beta_w,\beta_R,\gamma)\begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \\ t+1 \end{pmatrix} - \left(\tfrac{-1}{\gamma_{1,11}},\tfrac{\gamma_{1,12}}{\gamma_{1,11}},\tfrac{\gamma_{1,13}}{\gamma_{1,11}},\tfrac{\gamma_{1,14}}{\gamma_{1,11}}\right)\begin{pmatrix} E_t\Delta c_{t+1} \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix}
$$

$$
- \ldots - \left(\tfrac{\gamma_{5,11}}{\gamma_{1,11}},\tfrac{\gamma_{5,12}}{\gamma_{1,11}},\tfrac{\gamma_{5,13}}{\gamma_{1,11}},\tfrac{\gamma_{5,14}}{\gamma_{1,11}}\right)\begin{pmatrix} \Delta c_{t-4} \\ \Delta y_{t-4} \\ \Delta w_{t-4} \\ \Delta R_{t-4} \end{pmatrix} - \frac{1}{\gamma_{1,11}}(c'\vartheta + c'\Phi D_{t+1}).
$$

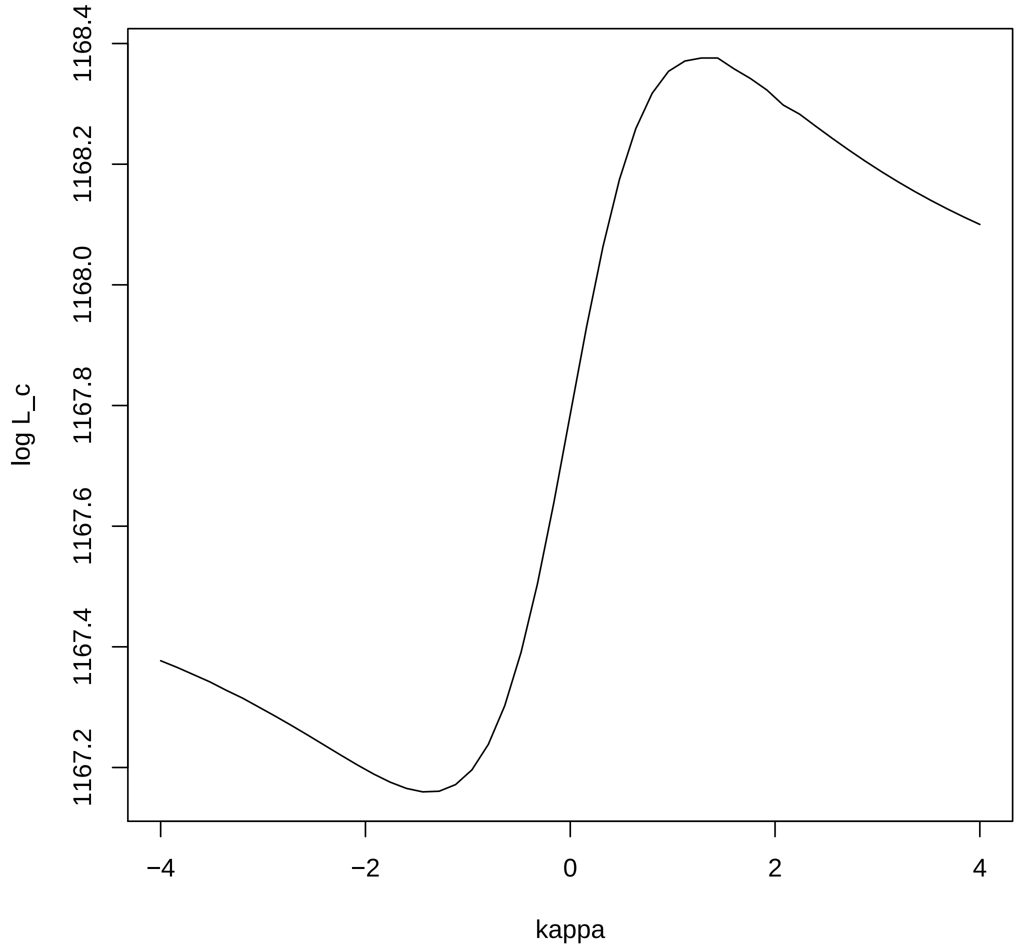
This formulation can be considered as a consumption function involving expected future consumption. The estimate of the coefficient of $E_t\Delta c_{t+1}$, $1/\gamma_{1,11}$ in (35), is of particular interest and equals $-1/0.41 = -2.44$ with estimated standard error of $(1/0.11)^2 = 82.6$. Thus, even if the estimate is outside the region where the parameter has an economically meaningful interpretation, the estimated standard error is so large that for all reasonable coefficients a confidence interval will cover such a region.

Thus far, we have only considered conditional expectations of future consumption in the CVAR. We now consider a more general formulation of the model involving conditional expectations of both future consumption and income to shed light on the magnitude of the proportion of rule of thumb consumers taking current income, and not future income, into account when determining the amount of consumption. Specifically, we consider a more general formulation of (34) expressed as

$$
\begin{aligned}
(38)\; E_t(\Delta c_{t+1} - \kappa \Delta y_{t+1}) \;&=\; \alpha_c(1, \beta_y, \beta_w, \beta_R, \gamma) \begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \\ t+1 \end{pmatrix} \\
&+(\gamma_{1,11}, \gamma_{1,12}, \gamma_{1,13}, \gamma_{1,14}) \begin{pmatrix} \Delta c_t \\ \Delta y_t \\ \Delta w_t \\ \Delta R_t \end{pmatrix} \\
&+ \ldots + (\gamma_{5,11}, \gamma_{5,12}, \gamma_{5,13}, \gamma_{5,14}) \begin{pmatrix} \Delta c_{t-4} \\ \Delta y_{t-4} \\ \Delta w_{t-4} \\ \Delta R_{t-4} \end{pmatrix} + \vartheta + \Phi D_{t+1},
\end{aligned}
$$

where the parameter $\kappa = \mu\lambda$, the proportion of the rule of thumb consumers that respond to changes in current income, is of particular interest. Accordingly, we consider a simplified version of (15) involving only κ and not all the economically interesting parameters $\mu, \phi, \lambda, \sigma, \tau$ and ϱ separately. After imposing $\gamma_{5,14} = \gamma_{5,13} = \gamma_{4,14} = 0$, in accordance with model 6 in Table 5, we can find the maximal log likelihood value for fixed values of κ. Figure 5 shows the concentrated log likelihood for κ in the region [-4,4]. The maximal value of around 1168.3 corresponds to the maximum likelihood estimate $\hat{\kappa} = 1.44$. The variation of the likelihood curve is less than 1.5 in magnitude over the whole range of $[-4, 4]$, meaning that a confidence interval with the customary confidence coefficients will cover this whole range and in particular the economically interesting values in the interval $[0, 1]$. Thus a large part of the confidence interval covers values of κ, which in light of (15), have no meaningful

Figure 5: Concentrated log likelihood for $\kappa = \mu\lambda$ in (38)[1]



Notes: Sample period: 1982q3−2008q3. [1]Model without a break in trend.

economic interpretation.

5.3 Estimation with a break in trend

As shown in Subsection 4.2, cointegration between consumption, income and wealth remains once a structural break around the financial crisis in 2008 is allowed for by the augmented CVAR in (26). Therefore, the empirical relevance of the augmented model with conditional expectations of future consumption and income will also be examined in the context of Johansen and Swensen (1999, 2004, 2008). We repeat (26) for convenience.

$$
\begin{aligned}
(39) \qquad \Delta X_t &= \alpha \begin{pmatrix} \beta \\ \gamma \end{pmatrix}' \begin{pmatrix} X_{t-1} \\ tSD_t \end{pmatrix} + \mu SD_t + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} \\
&\quad + \Phi D_t + \kappa_{2,1} ID_{2,t-1} + \ldots + \kappa_{2,k} ID_{2,t-k} + \epsilon_t,
\end{aligned}
$$

where $SD_t = (SD_{1,t}, SD_{2,t})'$, $\gamma = (\gamma_1', \gamma_2')'$ and $\mu = (\mu_1', \mu_2')'$. We now have that

$$
\begin{aligned}
E_t \Delta X_{t+1} &= \alpha\beta' X_t + \alpha\gamma' tSD_{t+1} + (\alpha\gamma' + \mu) SD_{t+1} \\
&+ \Gamma_1 \Delta X_t + \ldots + \Gamma_{k-1} \Delta X_{t-k+2} \\
&+ \Phi D_{t+1} + \kappa_{2,1} ID_{2,t} + \ldots + \kappa_{2,k} ID_{2,t-k+1}
\end{aligned}
$$

and

$$
\begin{aligned}
c' E_t \Delta X_{t+1} &= c'\alpha\beta' X_t + c'\alpha\gamma' tSD_{t+1} + c'(\alpha\gamma' + \mu) SD_{t+1} \\
&+ c'\Gamma_1 \Delta X_t + \ldots + c'\Gamma_{k-1} \Delta X_{t-k+2} \\
&+ c'\Phi D_{t+1} + c'\kappa_{2,1} ID_{2,t} + \ldots + c'\kappa_{2,k} ID_{2,t-k+1}.
\end{aligned}
$$

Table 6: Likelihood ratio test results for simplifying restrictions[1] on the augmented CVAR[2]

Model	Restrictions	$\log L_i$	$i-j^3$	$-2\log\frac{L_j}{L_i}$	df	p-value
1	-	1558.35	-	-	-	
2	$\beta_y+\beta_w=1$	1557.71	1-2	1.28	1	0.26
3	Model 2, $\gamma_{5,14}=0$	1557.69	2-3	0.04	1	0.84
4	Model 3, $\gamma_{5,13}=0$	1556.55	3-4	2.28	1	0.13
5	Model 4, $\gamma_{5,12}=0$	1554.87	4-5	3.36	1	0.07
6	Model 5, $\gamma_{5,11}=0$	1546.87	5-6	16.00	1	0.0001
7	Model 5, $\gamma_{4,14}=0$	1554.08	5-7	1.58	1	0.21
8	Model 7, $\gamma_{4,13}=0$	1550.13	7-8	7.86	1	0.0051
9	Model 7, $\gamma_{3,14}=0$	1554.06	7-9	0.04	1	0.84
10	Model 9, $\gamma_{2,14}=0$	1549.68	9-10	8.76	1	0.0031

Sample period: $1982q3-2016q4$. [1] See Johansen and Swensen (1999, 2004, 2008). [2] Model with a break in trend, see Johansen *et al.* (2000). [3] $i-j$ denotes the likelihood ratio test for the additional restriction(s) on model j compared to model i.

Inserting the restrictions from (31) for the non-deterministic terms yields

$$\begin{aligned} c'E_t\Delta X_{t+1} &= d'X_t + c'\alpha\gamma'(tSD_{t+1}+SD_{t+1}) + c'\mu SD_{t+1} \\ &- d'_{-1}\Delta X_t + \ldots - d'_{-k+1}\Delta X_{t-k+2} \\ &+ c'\Phi D_{t+1} + c'\kappa_{2,1}ID_{2,t} + \ldots + c'\kappa_{2,k}ID_{2,t-k+1}. \end{aligned}$$

Note that the coefficient matrix $c'\alpha\gamma'$ of $tSD_{t+1}+SD_{t+1}$ has reduced rank. It is now possible to proceed as before to see if any of the matrices $d'_{-1},\ldots,d'_{-k+1}$ can be deleted, by expanding X_t to also include $tSD_{t+1}+SD_{t+1}$.

Table 6 shows likelihood ratio tests for simplifying restrictions on the coefficients of the augmented CVAR considering conditional expectations of consumption over the sample period $1982q3-2016q4$. We end up with model 9 which includes five lags of consumption growth, four lags of both income and wealth growth and two lags of growth in the real interest rate. The estimated version of model 9 is displayed in (40) with estimated standard errors in parenthesis.

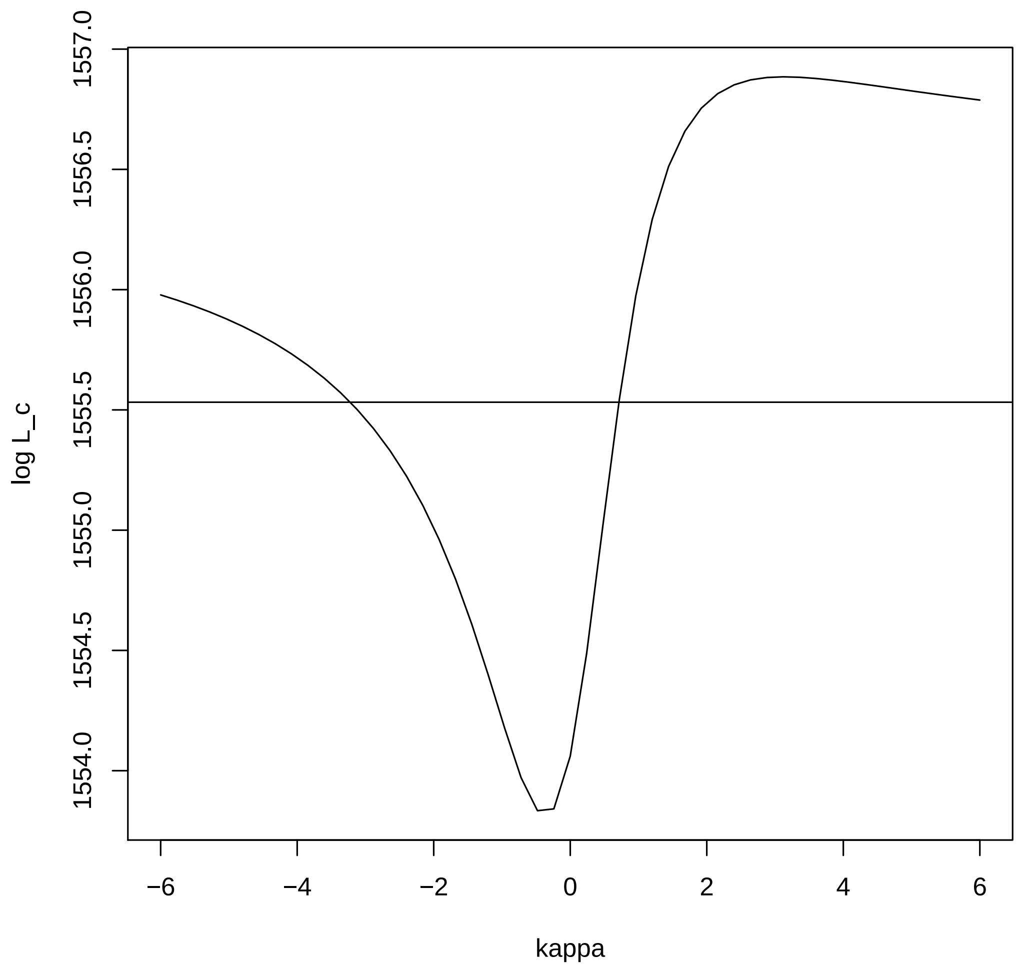
$$
(40) \qquad \widehat{E_t\Delta c_{t+1}} = -0.31(1.0, -0.84, -0.16, 1.78, 0.0008, 0.0049)\begin{pmatrix} c_t \\ y_t \\ w_t \\ R_t \\ tSD_{1,t+1} \\ tSD_{2,t+1} \end{pmatrix}
$$

$$
\begin{aligned}
&-\underset{(0.09)}{0.36}\Delta c_t - \underset{(0.10)}{0.12}\Delta c_{t-1} + \underset{(0.10)}{0.07}\Delta c_{t-2} + \underset{(0.10)}{0.50}\Delta c_{t-3} + \underset{(0.09)}{0.30}\Delta c_{t-4} \\
&-\underset{(0.12)}{0.28}\Delta y_t + \underset{(0.15)}{0.28}\Delta y_{t-1} + \underset{(0.15)}{0.08}\Delta y_{t-2} + \underset{(0.12)}{0.21}\Delta y_{t-3} \\
&+\underset{(0.67)}{0.23}\Delta w_t + \underset{(0.08)}{0.03}\Delta w_{t-1} - \underset{(0.08)}{0.05}\Delta w_{t-2} - \underset{(0.08)}{0.20}\Delta w_{t-3} \\
&+\underset{(0.23)}{0.51}\Delta R_t + \underset{(0.24)}{0.34}\Delta R_{t-1} + \textit{terms involving dummies.}
\end{aligned}
$$

The estimate of $1/\gamma_{1,11}$ is now $-1/0.36 = -2.78$ with a standard error of 7.72. Thus the conclusions from the case of no break in trend are maintained. Figure 6 plots the concentrated log likelihood for $\kappa = \mu\lambda$ in the region [-6,6] in a model similar to (38), but allowing for a break in the trend and the simplifying restrictions $\gamma_{5,14} = \gamma_{5,13} = \gamma_{5,12} = \gamma_{4,14} = \gamma_{3,14} = 0$ from model 9 in Table 6. The general shape of the concentrated likelihood is quite similar to what was found in the case without a trend break. The maximal value is now 1556.8 corresponding to the maximum likelihood estimator $\hat{\kappa} = 3.12$, which again in light of (15), does not make sense economically. However, the variation in the likelihood curve is much larger in this case. Also, the upper part of the 90 per cent confidence region, indicated by a straight line in Figure 6, begins around $\kappa = 0.7$. Hence, the economically sensible values of κ, [0.7,1.0], is contained in the confidence region. That said, the lower part of the confidence region contains negative values of κ, which contradict the underlying parameter value assumptions in (15).

We conclude from all the findings in this section that most of the parameters stemming from the class of Euler equations are not supported by the data when considering conditional expectations of consumption and income in CVAR models. Only habit formation in line with

Figure 6: Concentrated log likelihood for $\kappa = \mu\lambda$ in the augmented CVAR with a 90 per cent confidence interval[1]



Notes: Sample period: 1982q3−2016q4. [1]Model with a break in trend.

Smets and Wouters (2003) seems to play an important role in explaining the Norwegian consumer behaviour.

6 Conclusions

In this paper, we have formulated a general CVAR that nests both a class of consumption Euler equations and various Keynesian type consumption functions. Using likelihood-based methods and Norwegian data, we found evidence of cointegration between consumption, income and wealth once a structural break around the financial crisis is accounted for. That consumption cointegrates with both income and wealth and not only with income demonstrates the empirical irrelevance of a consumption Euler equation. More importantly, we found that consumption equilibrium corrects to changes in income and wealth and not that income equilibrium corrects to changes in consumption, as would be the case when an Euler equation is true. Finally, we found that most of the parameters stemming from the class of Euler equations are not corroborated by the data when considering conditional expectations of future consumption and income in CVAR models. Only habit formation, typically included in DSGE models, seems to be important in explaining the Norwegian consumer behaviour. Our preferred model is a dynamic Keynesian type consumption function with a first year MPC of around 25 per cent, which is in line with the empirical findings in the recent literature.

We have relied on a CVAR in which a structural break in the cointegration relationship between consumption, income and wealth around the event of the financial crisis has been accounted for by a broken trend. A possible interpretation may be that the broken trend reflects increased uncertainty and thus increased precautionary savings in the wake of the financial crisis. Another possibility is that the broken trend picks up some important effects of omitted variables necessary to explain the changed consumer behaviour after the financial crisis. For instance, we have neither included a variable capturing the changing credit conditions faced by households nor disaggregated the wealth variable into separate variables for liquid assets, illiquid assets, debt and housing. Such variables may be important in a CVAR to adequately pick up effects of the household financial accelerator on consumption in the wake of the financial crisis. We leave this issue for future work.

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Appendix 1. Data definitions and sources

The original data set used in Jansen (2013) as well as the extended data set are collected from Statistics Norway and Norges Bank, unless otherwise noted.

C_t: Consumption expenditures in households and ideal organisations excluding expenditures on health services and housing, fixed 2006 prices. Source: Statistics Norway.

Y_t: Households real disposable income, excluding equity income, defined as nominal income deflated by PC_t, the price deflator for total consumption expenditures in households and ideal organisations excluding expenditures on health services and housing (2006=1). Source: Statistics Norway.

W_t: Real household wealth defined as nominal household wealth (NW_t), the sum of financial and housing wealth, deflated by PC_t.

$NW_t = [L_{t-1}+ML_{t-1}+NL_{t-1}-CR_{t-1} +(PH/PC)_t a_t K_{t-1}]PC_t$, where L_t is household liquid assets (money stock and deposits), ML_t is household medium liquid assets (equity and bonds), NL_t is household non-liquid assets (insurance claims), CR_t is household debt to banks and other financial institutions, PH_t is housing price index (2006=1), a_t is the fraction of residential housing stock owned by households and K_t is the real value of the residential housing stock (fixed 2006 prices). All financial wealth components and the residential housing stock in the definition of NW_t are in fixed values and they refer to the end of period $t-1$. The residential housing stock is updated each quarter by adding the gross investments in housing capital (in real terms deflated by PH_t) and deducting 0.4 per cent depreciation per quarter. Source: Statistics Norway. Data for nominal household wealth for the period 1982q3 to 1992q3 are from Erlandsen and Nymoen (2008). These data are chained in 1992q3 with data from Statistics Norway.

R_t: Real after tax interest rate for households defined as $4 \cdot RLB_t(1-\tau_t)-CPI_t/CPI_{t-4}+1$, where RLB_t is average interest rate on households' bank loans, τ_t is marginal income tax rate faced by households and CPI_t is headline consumer price index (2006=1). Sources: Statistics Norway and Norges Bank.

Appendix 2. Equations for $\Delta\widehat{w}_t$ and $\Delta\widehat{R}_t$[21]

$$\begin{aligned}
\Delta\widehat{w}_t = {} & \underset{(0.15)}{0.25}\Delta c_{t-1} + \underset{(0.15)}{0.23}\Delta c_{t-2} + \underset{(0.15)}{0.48}\Delta c_{t-3} + \underset{(0.14)}{0.28}\Delta c_{t-4} + \underset{(0.12)}{0.26}\Delta c_{t-5} \\
& -\underset{(0.15)}{0.009}\Delta y_t - \underset{(0.21)}{0.20}\Delta y_{t-1} - \underset{(0.23)}{0.46}\Delta y_{t-2} - \underset{(0.22)}{0.39}\Delta y_{t-3} - \underset{(0.21)}{0.31}\Delta y_{t-4} \\
& -\underset{(0.17)}{0.18}\Delta y_{t-5} + \underset{(0.10)}{0.29}\Delta w_{t-1} - \underset{(0.11)}{0.09}\Delta w_{t-2} + \underset{(0.10)}{0.02}\Delta w_{t-3} + \underset{(0.10)}{0.15}\Delta w_{t-4} \\
& -\underset{(0.10)}{0.24}\Delta w_{t-5} - \underset{(0.35)}{0.13}\Delta R_{t-1} + \underset{(0.31)}{0.34}\Delta R_{t-2} + \underset{(0.32)}{0.10}\Delta R_{t-3} - \underset{(0.29)}{0.54}\Delta R_{t-4} \\
& +\underset{(0.31)}{0.41}\Delta R_{t-5} - \underset{(0.12)}{0.24}c_{t-1} + \underset{(0.10)}{0.20}y_{t-1} + \underset{(0.02)}{0.04}w_{t-1} - \underset{(0.20)}{0.46}R_{t-1} \\
& -\underset{(0.0001)}{0.0002}tSD_{1,t} - \underset{(0.00058)}{0.0012}tSD_{2,t} + terms\ involving\ dummies \\
& \hat{\sigma}_w = 0.024
\end{aligned}$$

$$\begin{aligned}
\Delta\widehat{R}_t = {} & \underset{(0.05)}{0.01}\Delta c_{t-1} + \underset{(0.05)}{0.21}\Delta c_{t-2} + \underset{(0.05)}{0.01}\Delta c_{t-3} + \underset{(0.04)}{0.03}\Delta c_{t-4} + \underset{(0.04)}{0.08}\Delta c_{t-5} \\
& +\underset{(0.04)}{0.06}\Delta y_t - \underset{(0.07)}{0.006}\Delta y_{t-1} - \underset{(0.07)}{0.02}\Delta y_{t-2} - \underset{(0.07)}{0.08}\Delta y_{t-3} - \underset{(0.06)}{0.09}\Delta y_{t-4} \\
& -\underset{(0.05)}{0.02}\Delta y_{t-5} - \underset{(0.03)}{0.04}\Delta w_{t-1} + \underset{(0.03)}{0.002}\Delta w_{t-2} - \underset{(0.03)}{0.03}\Delta w_{t-3} - \underset{(0.03)}{0.03}\Delta w_{t-4} \\
& +\underset{(0.03)}{0.009}\Delta w_{t-5} + \underset{(0.10)}{0.15}\Delta R_{t-1} + \underset{(0.09)}{0.14}\Delta R_{t-2} + \underset{(0.09)}{0.17}\Delta R_{t-3} - \underset{(0.08)}{0.27}\Delta R_{t-4} \\
& -\underset{(0.09)}{0.04}\Delta R_{t-5} - \underset{(0.05)}{0.15}c_{t-1} + \underset{(0.04)}{0.12}y_{t-1} + \underset{(0.01)}{0.03}w_{t-1} - \underset{(0.06)}{0.28}R_{t-1} \\
& -\underset{(0.0000)}{0.00013}tSD_{1,t} - \underset{(0.0002)}{0.0007}tSD_{2,t} + terms\ involving\ dummies \\
& \hat{\sigma}_R = 0.007
\end{aligned}$$

[21]Estimated standard errors in parenthesis.

Appendix 3. Details about the estimation procedure

The idea behind the estimation procedure outlined in Section 5 can be explained by means of the simple model $\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1}\Delta X_{t-k+1} + \vartheta + \epsilon_t$ with restrictions $c' E_t \Delta X_{t+1} - d' X_t + d'_{-1}\Delta X_t + \ldots + d'_{-k+1}\Delta X_{t-k+2} + \vartheta = 0$. Hence, there are no restrictions on the constant term, which must be estimated. The matrices $d, d_{-1}, \ldots, d_{-k+1}$ are, however, first considered as fixed. The variables are Gaussian. From Johansen and Swensen (1999) it follows that to find the maximum likelihood estimators one has to consider a conditional equation and a marginal equation. The likelihood is of the form $L_{1\cdot 2,max}(d, d_{-1}, \ldots, d_{-k+1}) L_{2,max}(d, d_{-1}, \ldots, d_{-k+1})$. The marginal equation takes the form

$$c'\Delta X_t = d' X_{t-1} - d'_{-1}\Delta X_{t-1} - \ldots - d'_{-k+2}\Delta X_{t-k+2} - d'_{-k+1}\Delta X_{t-k+1} + c'\vartheta + c'\epsilon_t.$$

The maximal value of the marginal likelihood can therefore be found by regressing $c'\Delta X_t - d' X_{t-1} + d'_{-1}\Delta X_{t-1} + \cdots + d'_{-k+1}\Delta X_{t-k+1}$ on $c'1$, where 1 is a $p \times 1$ vector, so $L_{2,max}(d, d_{-1}, \ldots, d_{-k+1})$ has a closed form. Let $c_\perp$ be a $p \times p - q$ matrix so $(c, c_\perp)$ has full rank and $c' c_\perp = 0$. and let $\bar{c} = c(c'c)^{-1}$. The conditional equation then takes the form

$$\begin{aligned} c'_\perp \Delta X_t &= \eta \xi' \bar{d}'_\perp X_{t-1} \\ &- \rho(c'\Delta X_{t-1} - d' X_{t-1} + d'_{-1}\Delta X_{t-1} + \ldots + d'_{-k+1}\Delta X_{t-k+1} - c'\vartheta) \\ &+ \theta\ (d'd)^{-1} d' X_{t-1} + c'_\perp \Gamma_1 \Delta X_{t-1} + \ldots + c'_\perp \Gamma_{k-1}\Delta X_{t-k+1} + c'_\perp \vartheta + u_t, \end{aligned}$$

where $u_t = (c'_\perp - \rho c')\epsilon_t$. For d and $d_{-1}, \ldots, d_{-k+1}$ fixed, the maximal values of the likelihood can be computed by reduced rank regression. The matrices η, ξ, $\rho = c'_\perp \Omega c (c'\Omega c)^{-1}$ and θ have dimensions $(p-q) \times (r-q)$, $(p-q) \times (r-q)$, $(p-q) \times q$ and $(p-q) \times q$, respectively.

Now we want to consider maximization over d, $d_{-1}, \ldots, d_{-k+1}$. Since these quantities occur in both the marginal and conditional equations the product $L_{1\cdot 2,max}(d, d_{-1}, \ldots$

$, d_{-k+1})L_{2,max}(d, d_{-1}, \ldots, d_{-k+1})$ must be considered. Using a generic numerical optimization procedure is an option, but the number of parameters quickly gets large. We therefore propose another procedure. If d is fixed, new values for $d_{-1}, \ldots, d_{-k+1}$ can be found by regressing $c'\Delta X_t - d'X_{t-1}$ on $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}$ and $c'1$. Because $d_{-1}, \ldots, d_{-k+1}$ also occur in the conditional equation it may be that also restrictions arising from this part must be taken into account. Reformulating the conditional equation as

$$\begin{aligned} c'_\perp \Delta X_t &= \eta\xi'\bar{d}'_\perp X_{t-1} \\ &- \rho(c'\Delta X_{t-1} - d'X_{t-1}) + \theta\ (d'd)^{-1}d'X_{t-1} \\ &+ (c'_\perp\Gamma_1 - \rho d'_{-1})\Delta X_{t-1} - \ldots + (c'_\perp\Gamma_{k-1} - \rho d'_{-k+1})\Delta X_{t-k+1} \\ &+ (c'_\perp - \rho c')\vartheta + u_t, \end{aligned}$$

one can see that there are no such constraints since $(c'_\perp\Gamma_1 - \rho d'_{-1}), \ldots, (c'_\perp\Gamma_{k-1} - \rho d'_{-k+1})$ and $(c'_\perp - \rho c')\vartheta$ vary freely and the parameters can be estimated by reduced rank regression.

Thus $L_{1\cdot 2,max}(d, d_{-1}, \ldots, d_{-k+1})L_{2,max}(d, d_{-1}, \ldots, d_{-k+1})$ is concentrated and depends only on the values in d. The maximum value can be found using a generic optimization procedure. As there are no restrictions on $d, d_1, \ldots, d_{k-1}$ this maximum will be the same as for the reduced rank VAR. Restrictions on $d_{-1}, \ldots, d_{-k+1}$, for instance $d_{-k+1} = d^0_{-k+1}$, can be treated as above, but regressing $c'\Delta X_t - d'X_{t-1} + d^0_{k-1}\Delta X_{t-k+1}$ on $d'_{-1}\Delta X_{t-1}, \ldots, d'_{-k+2}\Delta X_{t-k+2}$ and $c'1$. The conditional equation takes the form

$$\begin{aligned} c'_\perp \Delta X_t &= \eta\xi'\bar{d}'_\perp X_{t-1} \\ &- \rho(c'\Delta X_{t-1} - d'X_{t-1} + d^0_{k-1}\Delta X_{t-k+1}) + \theta\ (d'd)^{-1}d'X_{t-1} \\ &+ (c'_\perp\Gamma_1 - \rho d'_{-1})\Delta X_{t-1} + \ldots + (c'_\perp\Gamma_{k-2} - \rho d'_{-k+2})\Delta X_{t-k+2} \\ &+ c'_\perp\Gamma_{k-1}\Delta X_{t-k+1} + (c'_\perp - \rho c')\vartheta + u_t \end{aligned}$$

where the parameters can be estimated by reduced rank regression.